# NAVAL POSTGRADUATE SCHOOL

## Monterey, California

# THESIS

AN INVESTIGATION OF THE DYNAMIC MODEL
OF MODERN MILITARY CONFLICT

by

Timothy William Fox

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The Dynamic Model of Modern Military Conflict developed by Dr. Paul Moose, Naval Postgraduate School, is described by its system of differential equations followed by an investigation of its behavior. This investigation is predicated by an analysis of the model's stability about equilibrium using a method attributed to the study of ecosystems. The basis for this analysis is the formulation and subsequent evaluation of a community matrix termed


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Specific investigation of the model's behavior is observed varying the Command, Control, Communications and Intelligence ($C^3I$) enhancements of one combat force against that of an opposing force while one force utilizes misinformation and deception as a counter $C^3$ tactic and the other uses physical destruction as a counter $C^3$ tactic.

Potential military use of this model as an analytical tool for playing "what if?" type wargames is envisioned following further research and study of the effects on model behavior by varying other parameters not specifically addressed in this initial investigation.

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# An Investigation of the Dynamic Model of Modern Military Conflict

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# ABSTRACT

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

# I. INTRODUCTION

A military conflict has a variety of elements that potentially influence its eventual outcome. Commanders of the various echelons within the combat environment determined by a particular military conflict are faced with decisions that require the fusion of information from countless sources. These sources may be highly sophisticated sensors, relaying timely digital information concerning enemy force concentrations or movements, aircraft or missile tracks both friendly and nonfriendly, or perhaps intercepted communications signals. These sources could also be the professional schools, exercise participation or intelligence documents that have molded the commander's decision process over his military career whether it be extensive or minimal. Regardless of the case, decisions are made with some reliance on or rejection of prior knowledge. The assimulation of this prior knowledge, whether relative to the situation or not can be a monumental task in itself.

The sophistication of computer driven information systems and a presumed need for rapid decision making either by the man as a commander or the machine as his extension has caused a proliferation of model development in recent years [Ref. 1]. These models attempt to reflect the variety of elements that potentially influence the outcomes of military engagements, manipulate them according to some predetermined scheme or

algorithm and finally output some prescription or calculated result. The output of such models, together with the particular input elements and scheme of manipulation offer limitless applications either for direct command decision making, training or running countless "what if" exercises relative to known or suspected enemy and friendly combat information or characteristics.

## A. PURPOSE

The primary purpose of this thesis is to initiate investigation of one such model that exhibits the potential for use in a variety of areas, but in particular in looking at the "what if" questions associated with command, control and communications ($C^3$) and counter $C^3$. The specific model under investigation is the Dynamic Model of Modern Military Conflict proposed by Dr. Paul Moose, Academic Associate for the Joint Command, Control and Communications curricula at the Naval Postgraduate School [Ref. 2].

## B. SCOPE

The basic description of the model is given in Chapter 2. This description is in the form of a system of differential equations similar to that which describes Lanchester-type attrition models, however it includes terms relative to $C^3$ and counter $C^3$ for opposing forces. In this respect, information possessed by opposing forces is elevated to an equivalent level with the forces themselves. Furthermore, the description

includes a discussion on how the equations of this model are also similar to evolution equations which describe multi-species ecosystems. Such systems include natural death (loss) rates and replenishment of species within a population thus leading to a mathematical discussion of this model's behavior relative to an established equilibrium.

Following this description, Chapter 3 looks at the stability of the model in terms of a "community matrix", another reference to ecosystems which incorporates those parameters which tend to increase or reduce species within a population or community. This "community matrix" is renamed the "conflict matrix" for this model and explored by assigning a certain set of asymmetrical values to the elements of the matrix relative to hypothetical opposing forces in a military conflict. Specifically the investigation undertaken in this chapter, demonstrates conditions where opposing forces stabilize at a set equilibrium point or diverge from this point in unstable fashion.

Chapter 4 looks at more specific behaviors relative to the stability and instability demonstrated in Chapter 3. The identical initial conditions used in Chapter 3 are again used in this chapter and represent arbitrary units rather than specific calculated values. These values were chosen purely to demonstrate the general behavior of the model rather than replicate any specific military conflict.

Finally effects on equilibrium relative to the initial conditions used during stability and behavior investigations are explored in Chapter 5. Specifically, the initial conditions are aimed at isolating the effects of varying the command, control, communications and intelligence ($C^3I$) enhancements of one force against an opposing force's $C^3I$ enhancements. Utilizing these results, further demonstration of the model behavior is then achieved by varying resource and information variables associated with opposing military forces.

In essence, this thesis is concerned with an initial demonstration that the Dynamic Model of Modern Military Conflict can perform as might be expected rather than prove that it works in all cases. This approach opens several avenues to continued research leading to a more complete demonstration of its usefulness in evaluating the influence of $C^3I$ and counter $C^3$ in modern military conflict.

## II. A DYNAMIC MODEL OF MODERN MILITARY CONFLICT

### A. DEFINITION

The Dynamic Model of Modern Military Conflict incorporates the effects of $C^3I$ with basic attrition coefficients such as those found in Lanchester-type models. Specifically the effects of the above are described in terms of information ($I_x$ and $I_y$) and general purpose resources ($M_x$ and $M_y$) for opposing X and Y combat forces [Ref. 3]. The system of differential equations which characterize this model are given below followed by sections in this chapter which briefly touch on the background of the model and provide specific definitions of the terms involved.

(Eqn 2.1)

$$d(I_x)/dt = I_x[-A_x - \alpha_{xy}I_y - \gamma_{xy}M_y] + C_{xx}I_{xe}M_x + Q_xI_{xe}$$

(Eqn 2.2)

$$d(M_x)/dt = M_x[-B_x - \delta_{xy}I_y - \beta_{xy}M_y] - d_{xx}M_{xe}I_x - d_{xy}M_{xe}I_y - b_{xy}M_{xe}M_y + R_xM_{xe}$$

(Eqn 2.3)

$$d(I_y)/dt = I_y[-A_y - \alpha_{yx}I_x - \gamma_{yx}M_x] + C_{yy}I_{ye}M_y + Q_yI_{ye}$$

(Eqn 2.4)

$$d(M_y)/dt = M_y[-B_y - \delta_{yx}I_x - \beta_{yx}M_x] - d_{yy}M_{ye}I_y - d_{yx}M_{ye}I_x - b_{yx}M_{ye}M_x + R_yM_{ye}$$

## B. BACKGROUND

This system of four first order non-linear differential equations is of the form $\frac{d(\underset{\sim}{S})}{dt} = -\underset{\sim}{F}(\underset{\sim}{S}) + \underset{\sim}{\tilde{Q}}$. A set of equations in this form are known as evolution equations. $\underset{\sim}{S}$ and $\underset{\sim}{Q}$ are 4x1 column vectors and there are four functional relationships which are considered to be at most quadratic. Expanding the loss function term gives the following:

(Eqn 2.5)

$$- F_i(\underset{\sim}{S}) = -S_i\left[\sum_{j=1}^{4} \alpha_{ij} S_j\right] - \sum_{j=1}^{4} a_{ij} S_j, \quad i = 1,2,3,4$$

i.e.

$$- F_i(\underset{\sim}{S}) = - \text{quadrative term} - \text{linear term} \qquad \text{(Eqn 2.6)}$$

This system as it turns out is a very generalized form of the Lotka-Volterra equations for multi-species ecosystems [Ref. 4: 37-38]. More specifically it takes on the form:

(Eqn 2.7)

$$\frac{d(S_i)}{dt} = -S_i\left[\sum_{j=1}^{4} \alpha_{ij} S_j\right] - \sum_{j=1}^{4} a_{ij} S_j + \tilde{Q}_i, \quad i = 1,2,3,4$$

## C. EXPLANATION OF TERMS

In general the greek letter coefficients are quadratic, and the others are linear. Following is the definition of the terms that characterize the model:

$I_x$ - Information of force X

$I_{xe}$ - (Same as above at equilibrium)

$M_x$ - General purpose resources for force X

$M_{xe}$ - (Same as above at equilibrium)

$A_x$ - Natural death (loss) rate for $C^3I$ resources of force X

$B_x$ - Natural death (loss) rate for general purpose resources of force X

$\beta_{yx}$ - Lanchester "area fire" coefficient, X on Y

$b_{yx}$ - Lanchester "aimed fire" coefficient, X on Y

$Q_x$ - Fixed replenishment rate for $C^3I$ resources of force X

$R_x$ - Fixed replenishment rate for general purpose resources of force X

$C_{xx}$ - Replenishment from general purpose resources for $C^3I$ production, force X

$d_{xx}$ - Diminishment of X force for $C^3I$ production

$\alpha_{yx}$ - Counter $C^3$ by force X on force Y by misinformation and deception from $C^3I$ resources

$\gamma_{yx}$ - Counter $C^3$ by force X on force Y by physical destruction from general purpose resources

$\delta_{yx}$ - $C^3I$ (quadratic) force effectiveness enhancements, X on Y

$d_{yx}$ - $C^3I$ (linear) force effectiveness enhancements, X on Y

The remaining coefficients with y subscripts or xy subscripts refer to the Y force or Y on X activities.

## D. EXPECTED MODEL BEHAVIOR

The actual behavior of the model is dependent initially on the equilibrium for the system. One equilibrium point is determined by solving the following linear set of equations for $I_{xe}$, $M_{xe}$, $I_{ye}$ and $M_{ye}$:

(Eqn 2.8)

$$\begin{bmatrix} 0 & c_{xx} & -\alpha_{xy} & -\gamma_{xy} \\ -d_{xx} & 0 & -(\delta_{xy}+d_{xy}) & -(\beta_{xy}+b_{xy}) \\ -\alpha_{yx} & -\gamma_{yx} & 0 & c_{yy} \\ -(\delta_{yx}+d_{yx}) & -(\beta_{yx}+b_{yx}) & -d_{yy} & 0 \end{bmatrix} \begin{bmatrix} I_{xe} \\ M_{xe} \\ I_{ye} \\ M_{ye} \end{bmatrix} - \begin{bmatrix} A_x \\ B_x \\ A_y \\ B_y \end{bmatrix} + \begin{bmatrix} Q_x \\ R_x \\ Q_y \\ R_y \end{bmatrix} = \underset{\sim}{0}$$

Once equilibrium is established the effects of varying certain coefficients on dependent variables $I_x$, $M_x$, $I_y$, $M_y$ can be illustrated by solving the system of differential equations over the independent variable time.

# III. STABILITY ANALYSIS

## A. DEFINITION

Stability, as it relates to a deterministic model or deterministic system of equations, is more specifically defined as "neighborhood stability" or "stability in the vicinity of equilibirum". For population models with environmental parameters all well defined constants, interest centers on the community equilibrium where all species' populations have time-independent values, i.e. all net growth (death) rates are zero. Such an equilibrium may be called stable if, when the populations are perturbed, they in time return to their equilibrium values; the return may be achieved either as damped oscillations or monotonically. Conversely, if such a disturbance tends to amplify itself, the system is called unstable; such instability may appear as an oscillatory or as a monotonic growth in the disturbance. This definition is sufficient for linearized systems, however, it may be misleading for non-linear systems when full global stability of a system is of concern [Ref. 4:15].

Full global stability is said to be characterized by neighborhood analysis if the existence of a Lyapunov function can be determined [Ref. 4:15]. The existence of such a function is not explored in this thesis, however, regardless of whether such a function exists or not, neighborhood analysis

is within the scope of this thesis and founded in the generation of a community matrix.

## B. COMMUNITY MATRIX

The community matrix is said to both summarize the system (its elements being determined by the interactions between and within species near equilibrium) and sets the neighborhood stability by the sign of its eigenvalues (all negative real parts for all eigenvalues imply stability). Specifically it is generated by the following mathematical derivation [Ref. 4: 19-22].

Since the model is of the form

$$\frac{d(\underset{\approx}{S})}{dt} = -\underset{\sim}{F}(\underset{\sim}{S}) + \underset{\sim}{Q}$$

its multispecies population dynamics are given by the set of four equations:

$$\frac{d(S_i(t))}{dt} = F_i(S_1(t),\ S_2(t),\ S_3(t),\ S_4(t)),\quad i = 1,2,3,4 \qquad \text{(Eqn 3.1)}$$

The growth rate of the $i^{th}$ species at time $t$ is given by the nonlinear function $F_i$ of all relevant interacting populations. Equilibrium populations, $S_i^*$, follow from the 4 algebraic equations obtained by setting all growth rates at zero:

$$0 = F_i(S_1^*,\ S_2^*,\ S_3^*,\ S_4^*) \qquad \text{(Eqn 3.2)}$$

For each population, let

$$S_i(t) = S_i^* + s_i(t) \qquad \text{(Eqn 3.3)}$$

where $s_i$ measures the initially small perturbations to the $i^{th}$ population. Expanding each of the basic equations around this equilibrium in a Taylor series and discarding all second order and higher terms in the population perturbations $\underset{\sim}{s}$, a linearized approximation is obtained:

$$\frac{d\, s_i(t)}{dt} = \sum_{j=1}^{4} a_{ij}\, S_j(t) \qquad \text{(Eqn 3.4)}$$

Equivalently, in matrix notation

$$\frac{d\, X(t)}{dt} = A\, X(t) \qquad \text{(Eqn 3.5)}$$

where $X$ is the 4x1 column Matrix of the $s_i$, and $A$ is the four by four "community matrix" whose elements $a_{ij}$ describe the effect of species $j$ upon species $i$ near equilibrium. The elements $a_{ij}$ depend both on the details of the original equations and on the values of the equilibrium population through

$$a_{ij} = -\left(\frac{\partial F_i}{\partial S_j}\right)^* \qquad \text{(Eqn 3.6)}$$

The partial derivatives, denoting the derivatives of $F_i$ keeping all populations except $S_j$ constant, are then evaluated with all populations at their equilibrium values. The corresponding community matrix for the Dynamic Model of Modern Military Conflict is called the "conflict matrix" and its elements are given in the following matrix:

$$
\begin{bmatrix}
-(A_x + \alpha_{xy} + \gamma_{xy}) & C_{xx} & -\alpha_{xy} & -\gamma_{xy} \\
-d_{xx} & -(B_x+\delta_{xy}+\beta_{xy}) & -(\delta_{xy}+d_{xy}) & -(\beta_{xy}+b_{xy}) \\
-\alpha_{yx} & -\gamma_{yx} & -(A_y+\alpha_{yx}+\gamma_{yx}) & c_{yy} \\
-(\delta_{yx}+d_{yx}) & -(\beta_{yx}+b_{yx}) & -d_{yy} & -(B_y+\delta_{yx}+\beta_{yx})
\end{bmatrix}
$$

## C. METHOD OF ANALYSIS

To investigate the stability of the model, eigenvalues are computed relative to the conflict matrix given above and a set of initial values for the coefficients that comprise the elements of the matrix. Two cases are investigated: one in which element $d_{yx}$ is held constant and element $d_{xy}$ is varied across a range of values and another where $d_{xy}$ is held constant and $d_{yx}$ is varied across a range of values. The particular method of computing eigenvalues is the IMSL routine EIGRF [Ref. 5].

## D. INITIAL CONDITIONS

The initial conditions for investigating the two cases are given in Table I. These initial conditions were chosen to illustrate some of the potential behavior that this model may exhibit. The variable parameters are $C^3I$ enhancements for both X and Y forces. Furthermore the model is asymmetrical; X is using physical destruction by forces as a counter $C^3$ technique and Y is using misinformation and deception as a counter $C^3$ technique.

TABLE I

INITIAL CONDITIONS FOR STABILITY INVESTIGATION

$A_x = A_y = B_x = B_y = 0.5$

$c_{xx} = c_{yy} = d_{xx} = d_{yy} = 0.0$

$\beta_{xy} = \beta_{yx} = 1.0$

$\delta_{xy} = \delta_{yx} = 0.0$

$b_{xy} = b_{yx} = 0.0$

$\alpha_{xy} = 1.00$ (misinformation and deception)

$\alpha_{yx} = 0.0$

$\gamma_{xy} = 0.0$

$\gamma_{yx} = 1.0$ (physical destruction)

$d_{xy}$ = variable $C^3I$ enhancements (Y on X)

$d_{yx}$ = variable $C^3I$ enhancements (X on Y)

Equilibrium is taken to be the vector $\underset{\sim}{1}$ , i.e.

$I_{xe} = M_{xe} = I_{ye} = M_{ye} = 1$

Substitution of the initial parameter conditions into the conflict matrix gives the following matrix:

$$\begin{bmatrix} -1.5 & 0 & -1.0 & 0 \\ 0 & -1.5 & -d_{yx} & -1.0 \\ 0 & -1.0 & -1.5 & 0 \\ -d_{yx} & -1.0 & 0 & -1.5 \end{bmatrix}$$

The particular cases investigated then are:

1. $d_{yx}$ held constant at 1 and $d_{xy}$ varied from 0 to 2 by 0.05.
2. $d_{xy}$ held constant at 1 and $d_{yx}$ varied from 0 to 2 by 0.05.

## E. RESULTS AND DISCUSSION

The eigenvalues generated by case 1 are presented in Table II. It can be seen that for values of $d_{xy}$ greater than 0.80 a positive real eigenvalue appears. This is indicative of instability. Similarly all real parts of eigenvalues for $d_{xy}$ less than or equal 0.80 are negative thus indicating stability.

The eigenvalues generated by case 2 are presented in Table III. For values of $d_{yx}$ greater than 0.55 instability is indicated while stability is indicated for values of $d_{yx}$ less than or equal to 0.55.

TABLE II

EIGENVALUES FOR VARIABLE $C^3I$ ENHANCEMENTS (Y ON X)

| DXY | EIGEN 1 | EIGEN 2 | EIGEN 3 | EIGEN 4 |
|---|---|---|---|---|
| 2.00 | -1.500+0.550I | -1.500-0.550I | 0.317+0.000I | -3.317+0.000I |
| 1.95 | -1.500+0.554I | -1.500-0.554I | 0.305+0.000I | -3.305+0.000I |
| 1.90 | -1.500+0.558I | -1.500-0.558I | 0.292+0.000I | -3.292+0.000I |
| 1.85 | -1.500+0.562I | -1.500-0.562I | 0.279+0.000I | -3.279+0.000I |
| 1.80 | -1.500+0.566I | -1.500-0.566I | 0.266+0.000I | -3.266+0.000I |
| 1.75 | -1.500+0.570I | -1.500-0.570I | 0.254+0.000I | -3.254+0.000I |
| 1.70 | -1.500+0.574I | -1.500-0.574I | 0.241+0.000I | -3.241+0.000I |
| 1.65 | -1.500+0.579I | -1.500-0.579I | 0.228+0.000I | -3.228+0.000I |
| 1.60 | -1.500+0.583I | -1.500-0.583I | 0.215+0.000I | -3.215+0.000I |
| 1.55 | -1.500+0.588I | -1.500-0.588I | 0.202+0.000I | -3.202+0.000I |
| 1.50 | -1.500+0.592I | -1.500-0.592I | 0.188+0.000I | -3.188+0.000I |
| 1.45 | -1.500+0.597I | -1.500-0.597I | 0.175+0.000I | -3.175+0.000I |
| 1.40 | -1.500+0.602I | -1.500-0.602I | 0.162+0.000I | -3.162+0.000I |
| 1.35 | -1.500+0.607I | -1.500-0.607I | 0.149+0.000I | -3.149+0.000I |
| 1.30 | -1.500+0.612I | -1.500-0.612I | 0.135+0.000I | -3.135+0.000I |
| 1.25 | -1.500+0.617I | -1.500-0.617I | 0.122+0.000I | -3.122+0.000I |
| 1.20 | -1.500+0.622I | -1.500-0.622I | 0.108+0.000I | -3.108+0.000I |
| 1.15 | -1.500+0.627I | -1.500-0.627I | 0.095+0.000I | -3.095+0.000I |
| 1.10 | -1.500+0.632I | -1.500-0.632I | 0.081+0.000I | -3.081+0.000I |
| 1.05 | -1.500+0.638I | -1.500-0.638I | 0.067+0.000I | -3.067+0.000I |
| 1.00 | -1.500+0.644I | -1.500-0.644I | 0.054+0.000I | -3.054+0.000I |
| 0.95 | -1.500+0.649I | -1.500-0.649I | 0.040+0.000I | -3.040+0.000I |
| 0.90 | -1.500+0.655I | -1.500-0.655I | 0.026+0.000I | -3.026+0.000I |
| 0.85 | -1.500+0.661I | -1.500-0.661I | 0.012+0.000I | -3.012+0.000I |
| 0.80 | -1.500+0.667I | -1.500-0.667I | -0.002+0.000I | -2.998+0.000I |
| 0.75 | -1.500+0.674I | -1.500-0.674I | -0.015+0.000I | -2.985+0.000I |
| 0.70 | -1.500+0.680I | -1.500-0.680I | -0.029+0.000I | -2.971+0.000I |
| 0.65 | -1.500+0.687I | -1.500-0.687I | -0.044+0.000I | -2.956+0.000I |
| 0.60 | -1.500+0.693I | -1.500-0.693I | -0.058+0.000I | -2.942+0.000I |
| 0.55 | -1.500+0.700I | -1.500-0.700I | -0.072+0.000I | -2.928+0.000I |
| 0.50 | -1.500+0.707I | -1.500-0.707I | -0.086+0.000I | -2.914+0.000I |
| 0.45 | -1.500+0.714I | -1.500-0.714I | -0.100+0.000I | -2.900+0.000I |
| 0.40 | -1.500+0.722I | -1.500-0.722I | -0.114+0.000I | -2.886+0.000I |
| 0.35 | -1.500+0.729I | -1.500-0.729I | -0.128+0.000I | -2.872+0.000I |
| 0.30 | -1.500+0.737I | -1.500-0.737I | -0.143+0.000I | -2.857+0.000I |
| 0.25 | -1.500+0.744I | -1.500-0.744I | -0.157+0.000I | -2.843+0.000I |
| 0.20 | -1.500+0.752I | -1.500-0.752I | -0.171+0.000I | -2.829+0.000I |
| 0.15 | -1.500+0.761I | -1.500-0.761I | -0.185+0.000I | -2.815+0.000I |
| 0.10 | -1.500+0.769I | -1.500-0.769I | -0.200+0.000I | -2.800+0.000I |
| 0.05 | -1.500+0.777I | -1.500-0.777I | -0.214+0.000I | -2.786+0.000I |
| 0.00 | -1.500+0.786I | -1.500-0.786I | -0.228+0.000I | -2.772+0.000I |

TABLE III

EIGENVALUES FOR VARIABLE $C^3I$ ENHANCEMENTS (X ON Y)

| DYX | EIGEN 1 | EIGEN 2 | EIGEN 3 | EIGEN 4 |
|---|---|---|---|---|
| 2.00 | -3.153+0.000I | -1.500+0.856I | -1.500-0.856I | 0.153+0.000I |
| 1.95 | -3.149+0.000I | -1.500+0.847I | -1.500-0.847I | 0.149+0.000I |
| 1.90 | -3.144+0.000I | -1.500+0.838I | -1.500-0.838I | 0.144+0.000I |
| 1.85 | -3.140+0.000I | -1.500+0.830I | -1.500-0.830I | 0.140+0.000I |
| 1.80 | -3.135+0.000I | -1.500+0.821I | -1.500-0.821I | 0.135+0.000I |
| 1.75 | -3.130+0.000I | -1.500+0.811I | -1.500-0.811I | 0.130+0.000I |
| 1.70 | -3.126+0.000I | -1.500+0.802I | -1.500-0.802I | 0.126+0.000I |
| 1.65 | -3.121+0.000I | -1.500+0.792I | -1.500-0.792I | 0.121+0.000I |
| 1.60 | -3.116+0.000I | -1.500+0.783I | -1.500-0.783I | 0.116+0.000I |
| 1.55 | -3.111+0.000I | -1.500+0.773I | -1.500-0.773I | 0.111+0.000I |
| 1.50 | -3.107+0.000I | -1.500+0.762I | -1.500-0.762I | 0.107+0.000I |
| 1.45 | -3.102+0.000I | -1.500+0.752I | -1.500-0.752I | 0.102+0.000I |
| 1.40 | -3.097+0.000I | -1.500+0.741I | -1.500-0.741I | 0.097+0.000I |
| 1.35 | -3.092+0.000I | -1.500+0.730I | -1.500-0.730I | 0.092+0.000I |
| 1.30 | -3.086+0.000I | -1.500+0.719I | -1.500-0.719I | 0.086+0.000I |
| 1.25 | -3.081+0.000I | -1.500+0.707I | -1.500-0.707I | 0.081+0.000I |
| 1.20 | -3.076+0.000I | -1.500+0.695I | -1.500-0.695I | 0.076+0.000I |
| 1.15 | -3.070+0.000I | -1.500+0.683I | -1.500-0.683I | 0.070+0.000I |
| 1.10 | -3.065+0.000I | -1.500+0.670I | -1.500-0.670I | 0.065+0.000I |
| 1.05 | -3.059+0.000I | -1.500+0.657I | -1.500-0.657I | 0.059+0.000I |
| 1.00 | -3.054+0.000I | -1.500+0.644I | -1.500-0.644I | 0.054+0.000I |
| 0.95 | -3.048+0.000I | -1.500+0.630I | -1.500-0.630I | 0.048+0.000I |
| 0.90 | -3.042+0.000I | -1.500+0.615I | -1.500-0.615I | 0.042+0.000I |
| 0.85 | -3.036+0.000I | -1.500+0.600I | -1.500-0.600I | 0.036+0.000I |
| 0.80 | -3.030+0.000I | -1.500+0.585I | -1.500-0.585I | 0.030+0.000I |
| 0.75 | -3.024+0.000I | -1.500+0.568I | -1.500-0.568I | 0.024+0.000I |
| 0.70 | -3.018+0.000I | -1.500+0.551I | -1.500-0.551I | 0.018+0.000I |
| 0.65 | -3.011+0.000I | -1.500+0.533I | -1.500-0.533I | 0.011+0.000I |
| 0.60 | -3.005+0.000I | -1.500+0.515I | -1.500-0.515I | 0.005+0.000I |
| 0.55 | -2.998+0.000I | -1.500+0.495I | -1.500-0.495I | -0.002+0.000I |
| 0.50 | -2.992+0.000I | -1.500+0.474I | -1.500-0.474I | -0.008+0.000I |
| 0.45 | -2.985+0.000I | -1.500+0.452I | -1.500-0.452I | -0.015+0.000I |
| 0.40 | -2.978+0.000I | -1.500+0.428I | -1.500-0.428I | -0.022+0.000I |
| 0.35 | -2.970+0.000I | -1.500+0.402I | -1.500-0.402I | -0.030+0.000I |
| 0.30 | -2.963+0.000I | -1.500+0.374I | -1.500-0.374I | -0.037+0.000I |
| 0.25 | -2.955+0.000I | -1.500+0.344I | -1.500-0.344I | -0.045+0.000I |
| 0.20 | -2.948+0.000I | -1.500+0.309I | -1.500-0.309I | -0.052+0.000I |
| 0.15 | -2.940+0.000I | -1.500+0.269I | -1.500-0.269I | -0.060+0.000I |
| 0.10 | -2.931+0.000I | -1.500+0.221I | -1.500-0.221I | -0.069+0.000I |
| 0.05 | -2.923+0.000I | -1.500+0.157I | -1.500-0.157I | -0.077+0.000I |
| 0.00 | -2.914+0.000I | -1.500+0.000I | -1.500+0.000I | -0.086+0.000I |

The results of the above two cases are summarized graphically in Figures 1, 2, and 3. Figure 1 depicts the generation of eigenvalues with positive real parts (instability) beyond 0.5 and 0.8 for $d_{yx}$ and $d_{yx}$ respectively, and negative real parts elsewhere (stability). Figure 2 represents the other real eigenvalue which is always negative thus not contributing to instability. Figure 3 represents the positive imaginary part of the remaining two eigenvalues which turn out to be complex conjugate pairs with real part equal to -1.5. Existence of these imaginary parts indicate that the system does in fact oscillate while dampening out.

The eigenvalues generated by these two cases lead to more specific investigation of stability and instability. The eigenvalues have located the regions of stability and instability so now by using the appropriate values of $d_{xy}$ and $d_{yx}$ the behavior of the model can be observed relative to perturbations of the dependent variables over time for stable and unstable cases.

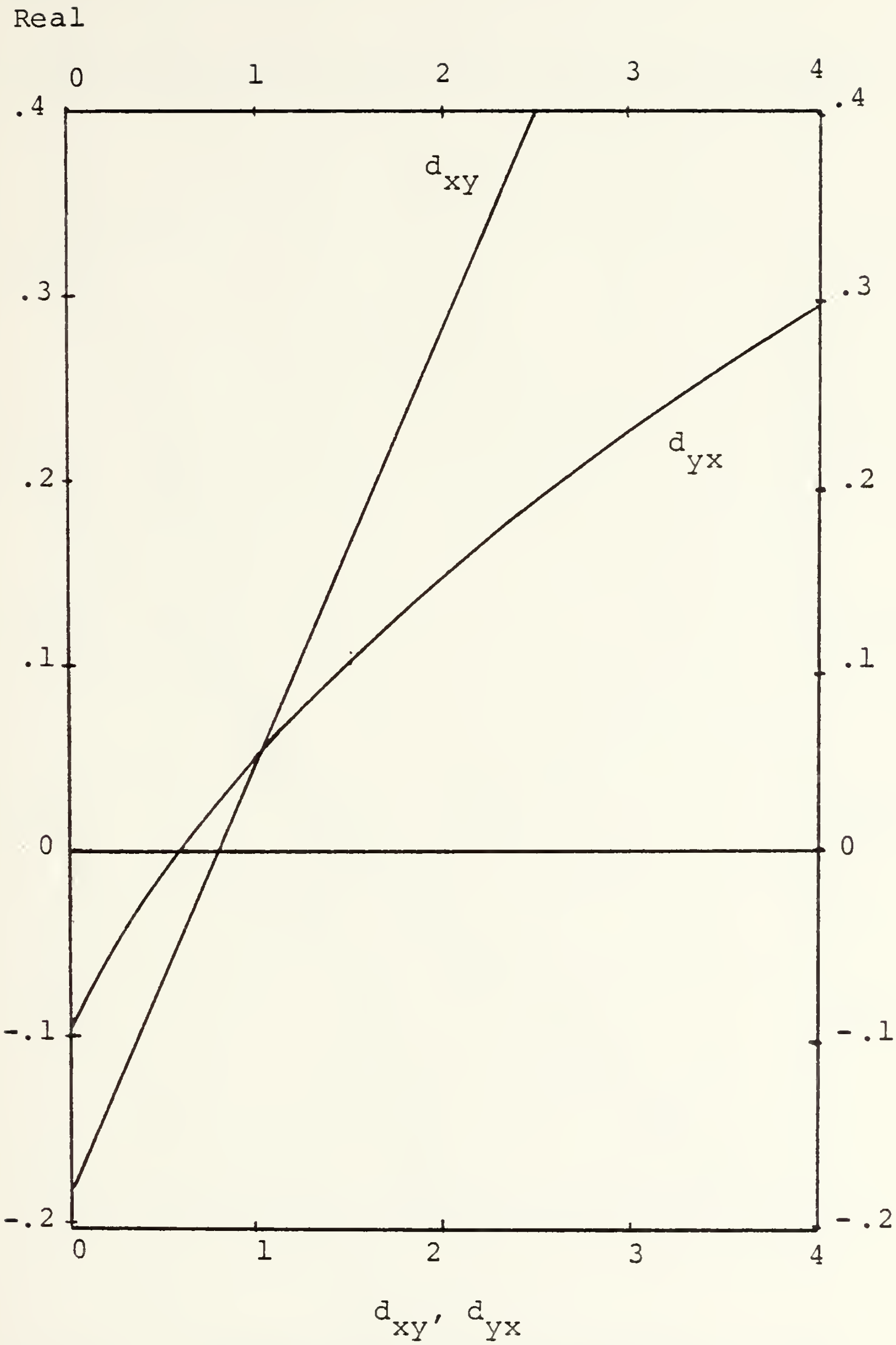


Figure 1. Eigenvalues with positive real parts (imaginary part = 0).

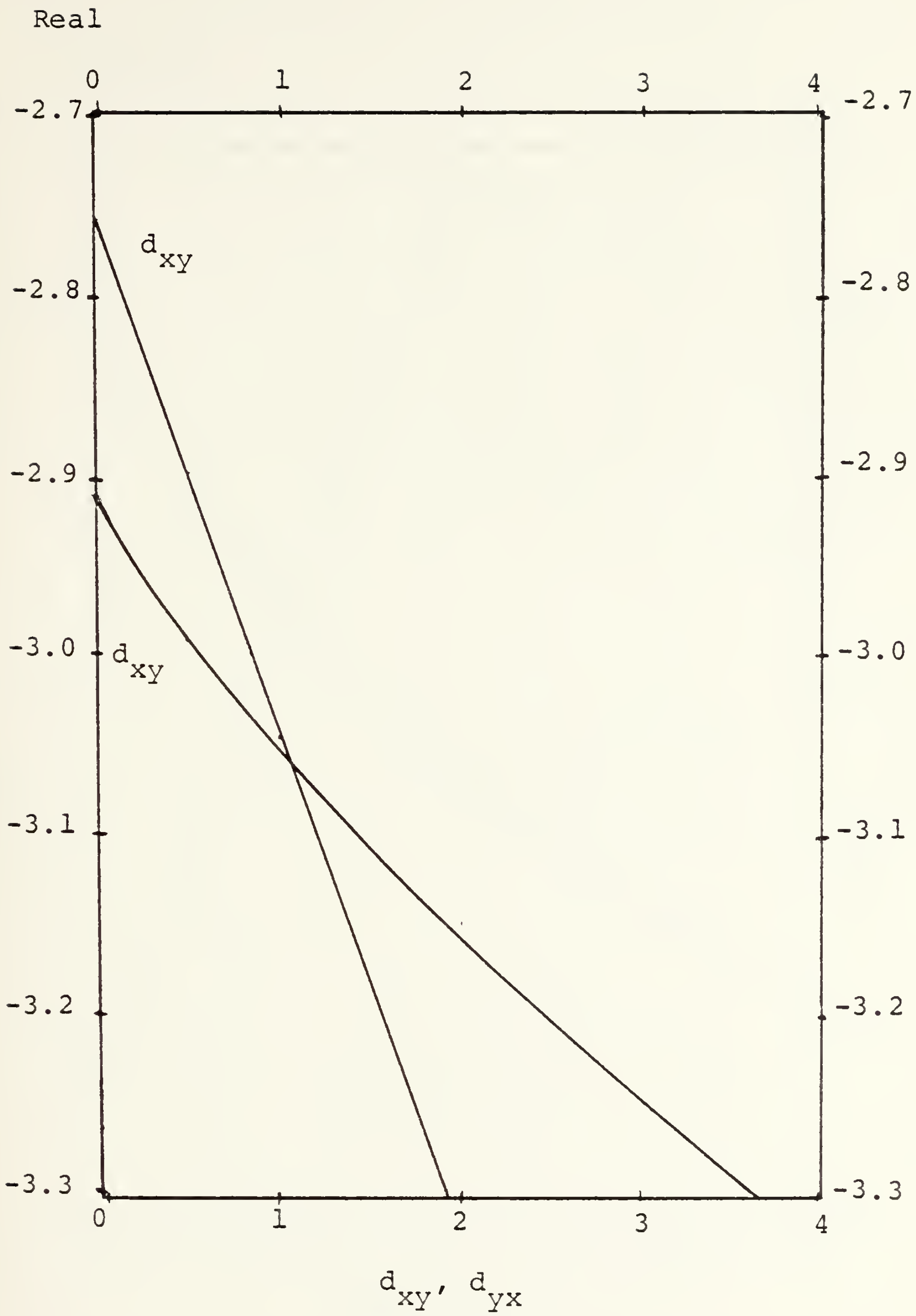


Figure 2. Eigenvalues with all negative real parts (imaginary part = 0).

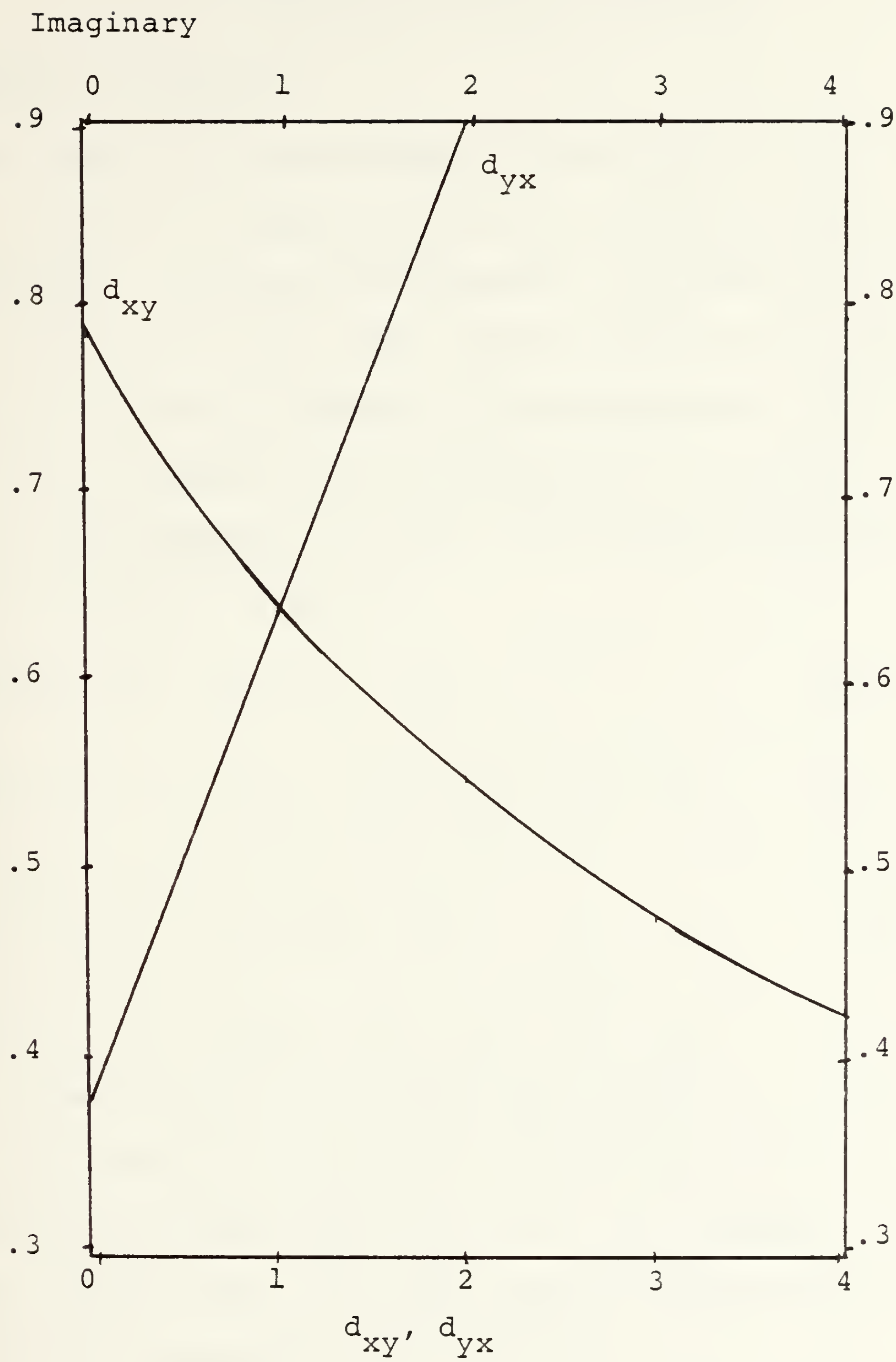


Figure 3. Positive imaginary parts of complex conjugate pairs (real part = -1.5).

# IV. SPECIFIC STABILITY ANALYSIS

## A. METHOD OF ANALYSIS

More specific illustrations of stability in the Dynamic Model of Modern Military Conflict are achieved by perturbing the dependent variables and observing the behavior of the model relative to time. The initial values used in the stability analysis of Chapter 3 are substituted into the differential equations which characterize the model. These differential equations turn out to be the following for the parameter values of Table I.

$$\frac{d\,I_x}{dt} = 1.5 - I_x\,(I_y + 0.5) \qquad \text{(Eqn 4.1)}$$

$$\frac{d\,M_x}{dt} = 1.5 - d_{xy}I_y - M_x\,(M_y + 0.5) + d_{xy} \qquad \text{(Eqn 4.2)}$$

$$\frac{d\,I_y}{dt} = 1.5 - I_y\,(M_x + 0.5) \qquad \text{(Eqn 4.3)}$$

$$\frac{d\,M_y}{dt} = 1.5 - d_{yx}I_x - M_y\,(M_x + 0.5) + d_{yx} \qquad \text{(Eqn 4.4)}$$

The Interactive Ordinary Differential Equations package (IODE) was employed using these differential equations and representative values for $d_{xy}$ and $d_{yx}$ (stable and unstable values) [Ref. 6]. Specifically, plots were generated to graphically represent two behaviors:

1. Dependent variables against the independent variable i.e., $I_x$, $M_x$, $I_y$, $M_y$ against time.
2. Phase plots of opposing forces i.e., $M_x$ against $M_y$.

## B. STABILITY WITH $d_{xy}$ AT 0.4

Table IV shows the initial values for the dependent variables, time duration and increment for plotting for each of fourteen trial runs of this model. Both $d_{xy}$ and $d_{yx}$ were held constant at 0.4 and 1.0 respectively. In addition, the final column in Table IV represents the time at which all dependent variables damped out to or very nearly to the established equilibrium of 1.

Graphically, Figure 4 depicts the general plot where only one dependent variable was perturbed (trials 1 through 4). Figures 5 through 8 further illustrate phase plots for these same trials of $M_x$ vs $M_y$ illustrating return to equilibrium over time.

Two other trials are illustrated in Figures 9 through 12. Specifically Figures 9 and 10 represent perturbations of both $I_x$ and $I_y$ (trial 6). The interesting phase plot in Figure 10 is indicative of the oscillatory nature of both $M_x$ and $M_y$ relative to these perturbations. Figure 11 depicts a triple perturbation involving $I_x$, $I_y$ and $M_y$, with the resulting oscillatory nature of $M_x$ and $M_y$ shown in Figure 12 (Trial 13).

Although the above illustrations are not all inclusive, it should be noted that all dependent variables did in fact damp out over the set time period and phase plots of $M_x$ and $M_y$ revealed return to equilibrium in all trials.

TABLE IV

INITIAL CONDITIONS WITH
$d_{xy}$ AT 0.4 AND $d_{yx}$ AT 1.0 (STABLE)

| Trial # | Time | Increment | $I_x$ | $M_x$ | $I_y$ | $M_y$ | Time for 90% Damp out (sec) |
|---|---|---|---|---|---|---|---|
| 1 | 0 to 50 | .05 | 1.25 | 1.00 | 1.00 | 1.00 | 7 |
| 2 | 0 to 50 | .05 | 1.00 | 1.25 | 1.00 | 1.00 | 12 |
| 3 | 0 to 50 | .05 | 1.00 | 1.00 | 1.25 | 1.00 | 9 |
| 4 | 0 to 50 | .05 | 1.00 | 1.00 | 1.00 | 1.25 | 10 |
| 5 | 0 to 50 | .05 | 1.25 | 1.25 | 1.00 | 1.00 | 17 |
| 6 | 0 to 50 | .05 | 1.25 | 1.00 | 1.25 | 1.00 | 1 |
| 7 | 0 to 50 | .05 | 1.25 | 1.00 | 1.00 | 1.25 | 1.5 |
| 8 | 0 to 50 | .05 | 1.00 | 1.25 | 1.00 | 1.25 | 1.5 |
| 9 | 0 to 50 | .05 | 1.00 | 1.00 | 1.25 | 1.25 | 15.5 |
| 10 | 0 to 50 | .05 | 1.00 | 1.25 | 1.25 | 1.00 | 4.0 |
| 11 | 0 to 50 | .05 | 1.25 | 1.25 | 1.25 | 1.00 | 12.0 |
| 12 | 0 to 50 | .05 | 1.25 | 1.25 | 1.00 | 1.25 | 11.0 |
| 13 | 0 to 50 | .05 | 1.25 | 1.00 | 1.25 | 1.25 | 12.0 |
| 14 | 0 to 50 | .05 | 1.00 | 1.25 | 1.25 | 1.25 | 4.0 |

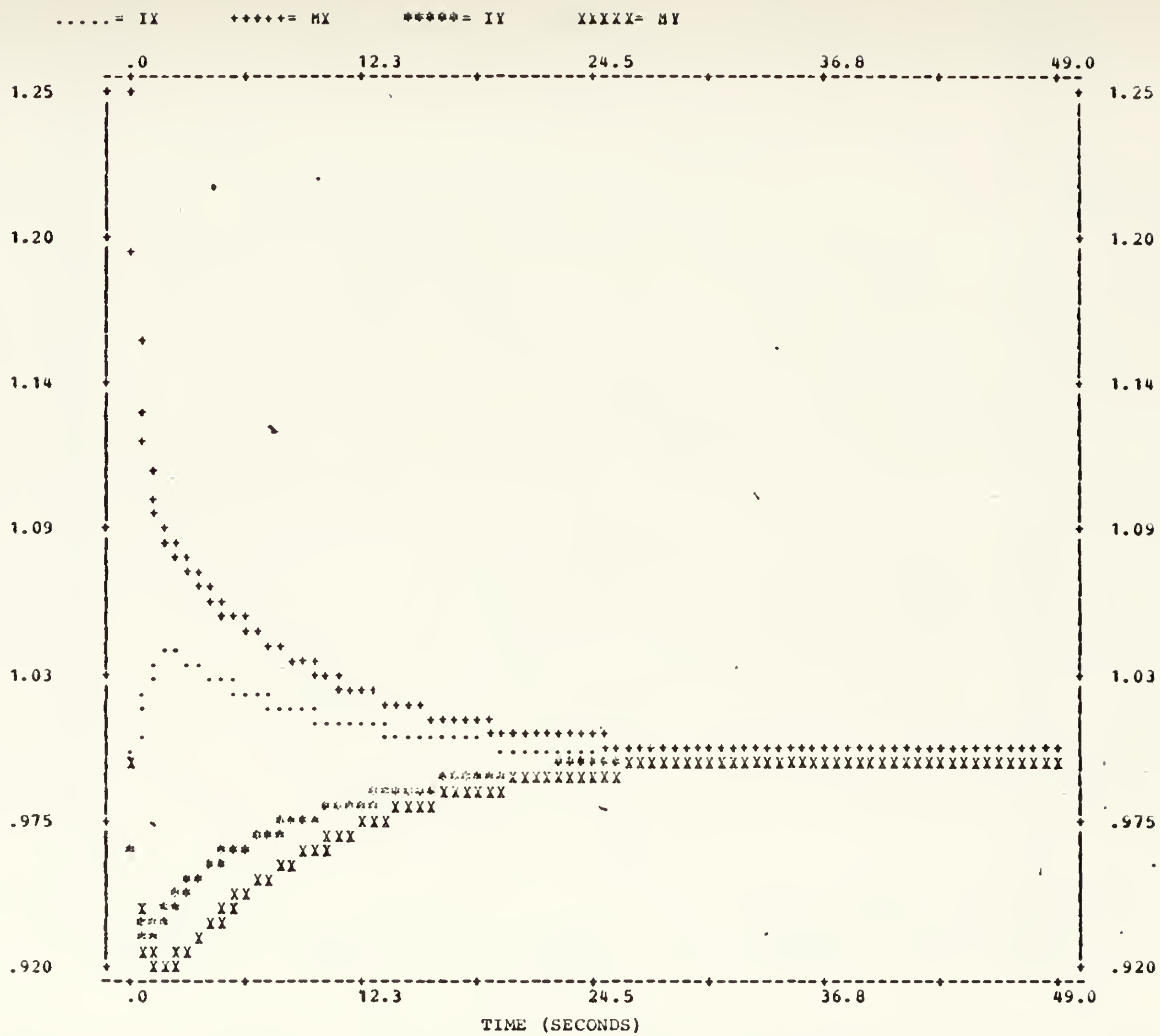


Figure 4. Single Perturbation, stable (general plot).

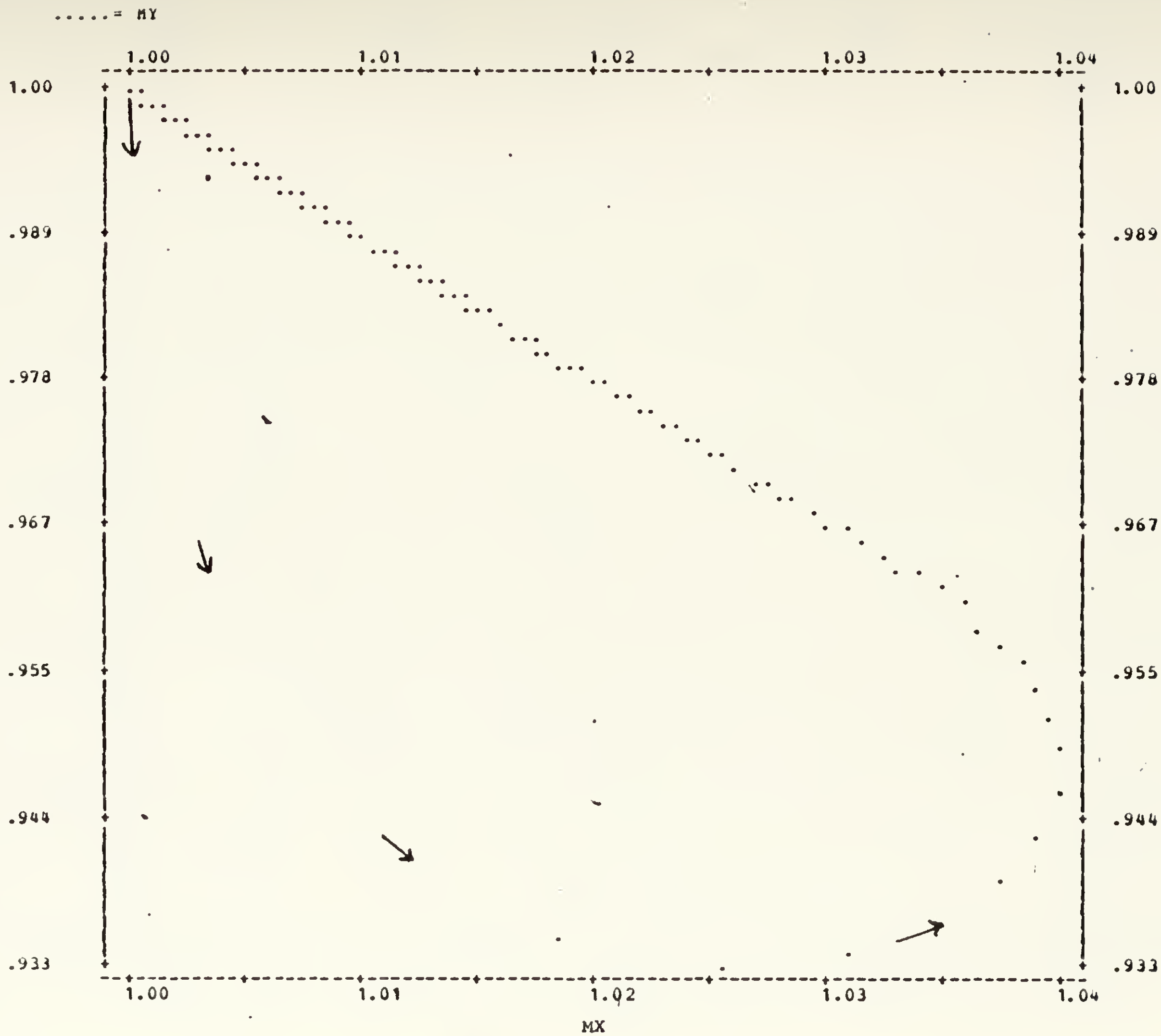


Figure 5. Single Perturbation, stable phase plot ($I_X$ at 1.25)

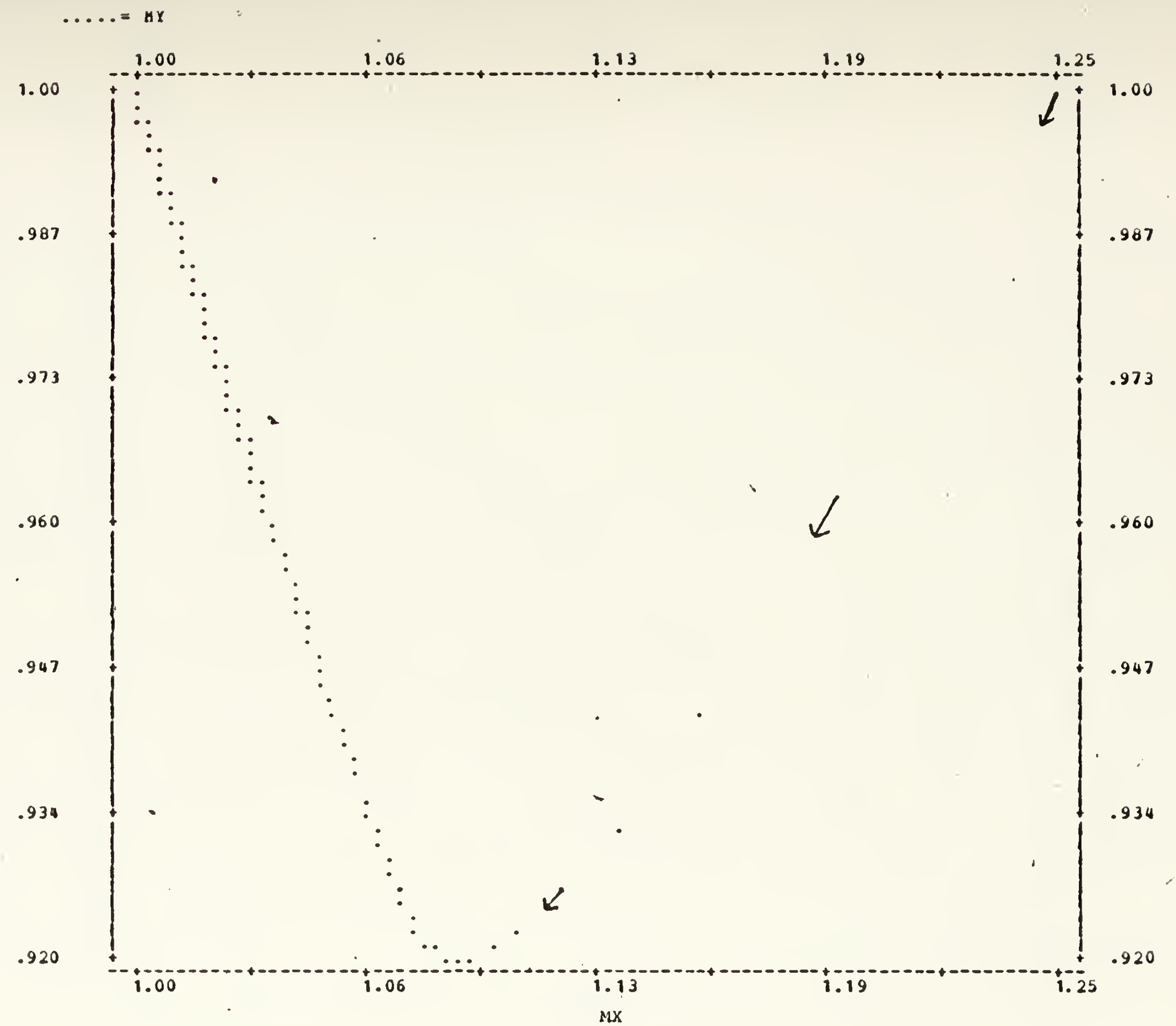


Figure 6. Single Perturbation, stable phase plot ($M_x$ at 1.25).

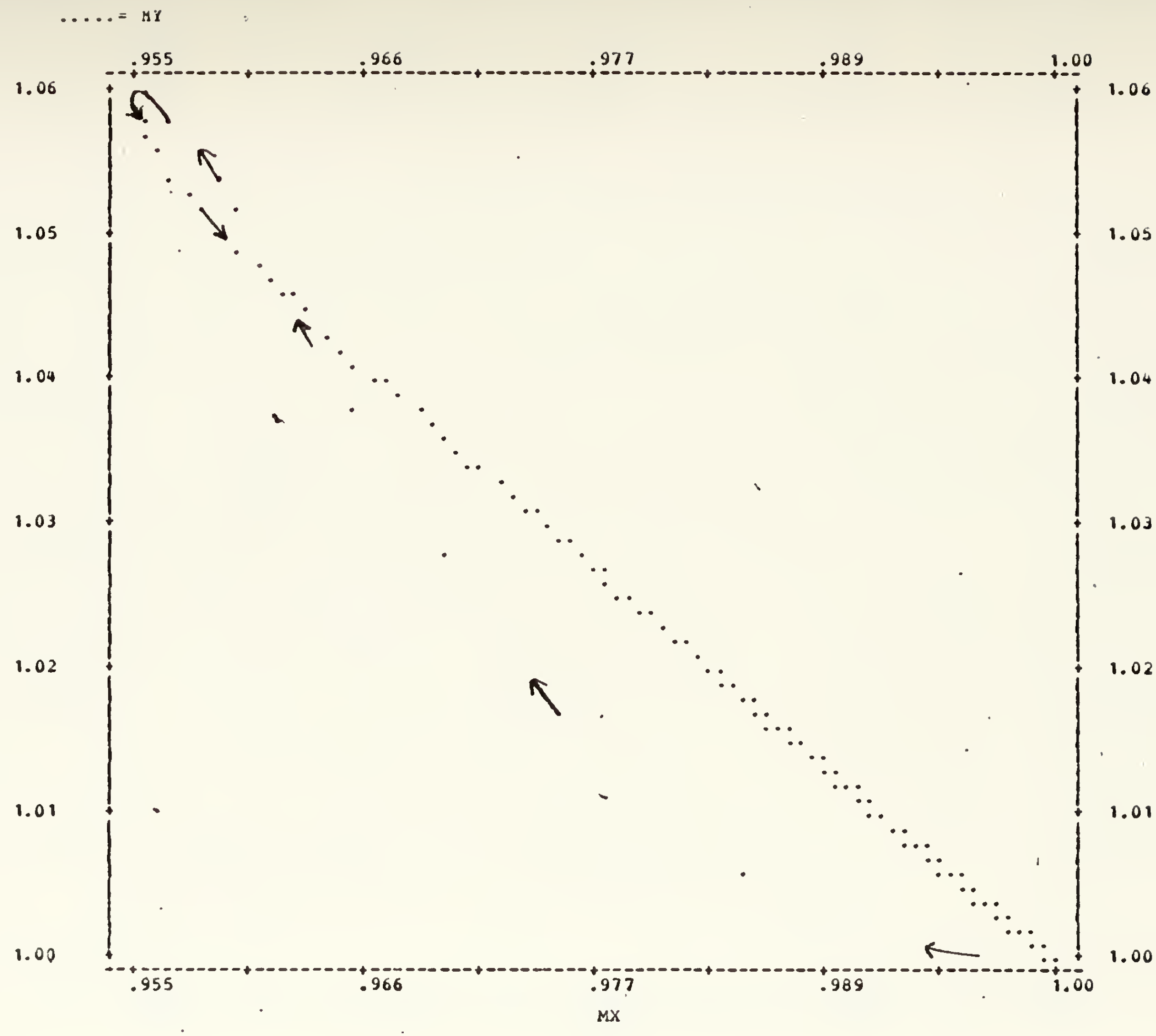


Figure 7. Single Perturbation, stable phase plot ($I_y$ at 1.25).

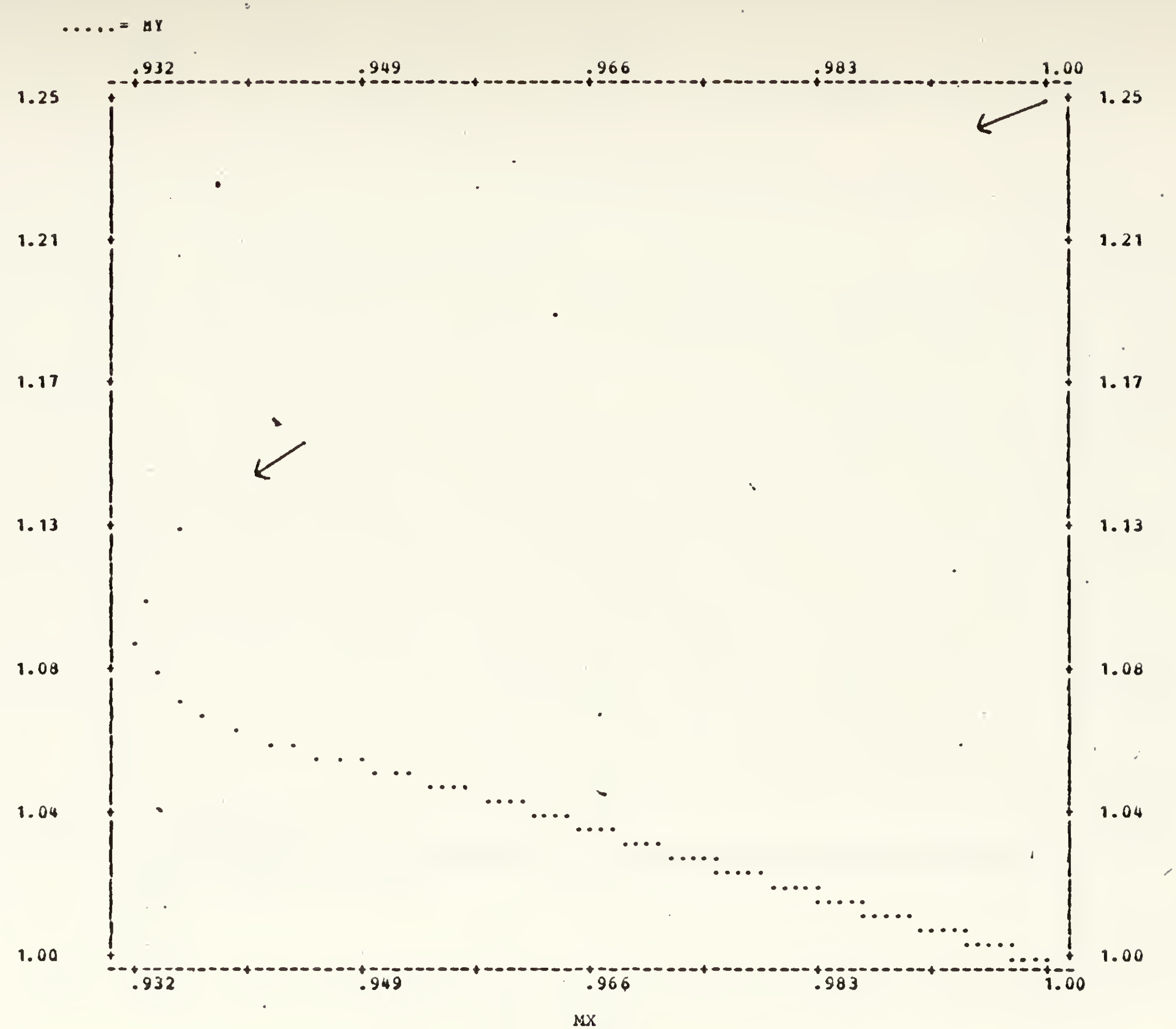


Figure 8. Single Perturbation, stable phase plot ($M_y$ at 1.25).

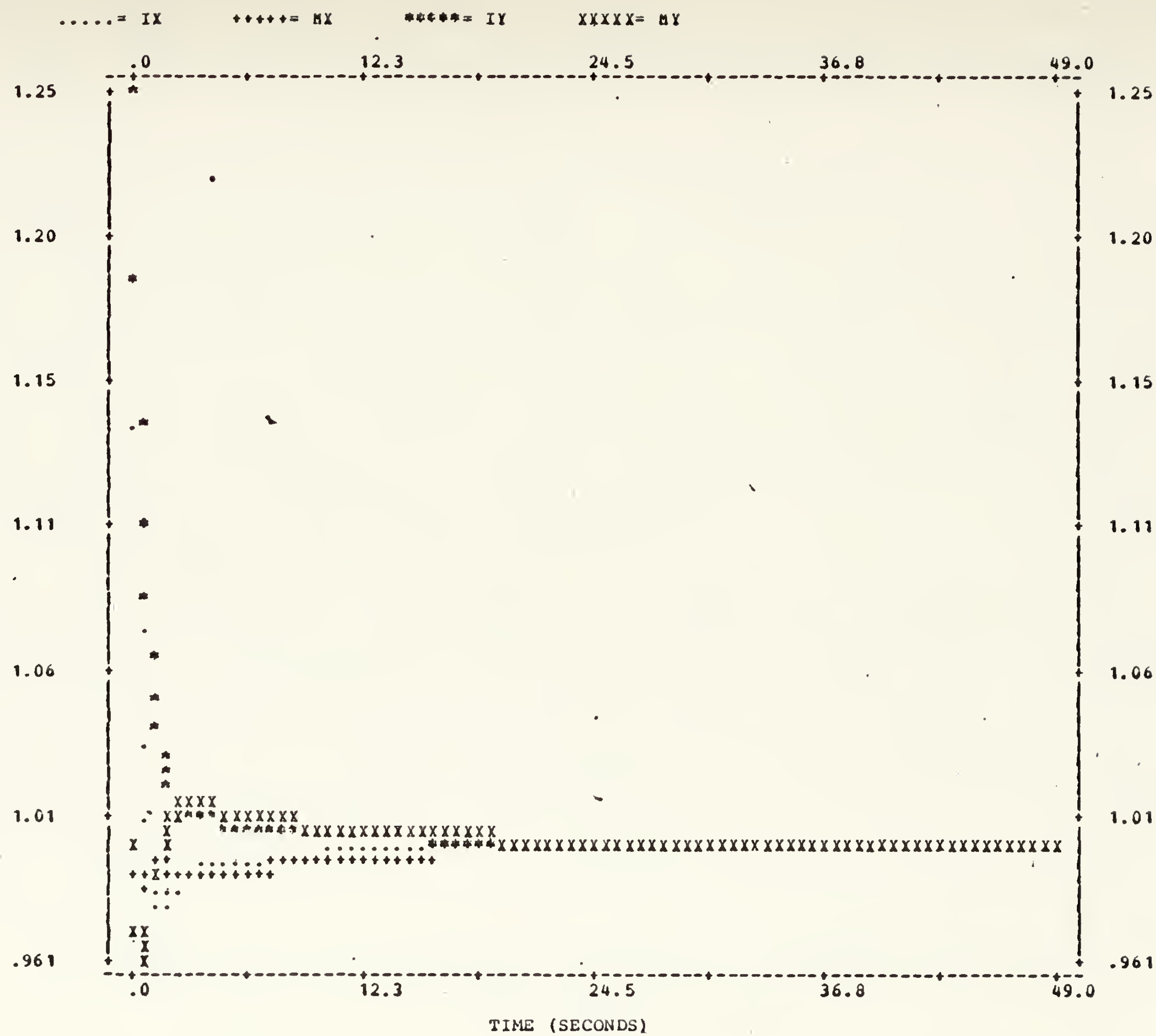


Figure 9. Double Perturbation, stable ($I_x$ and $I_y$ at 1.25).

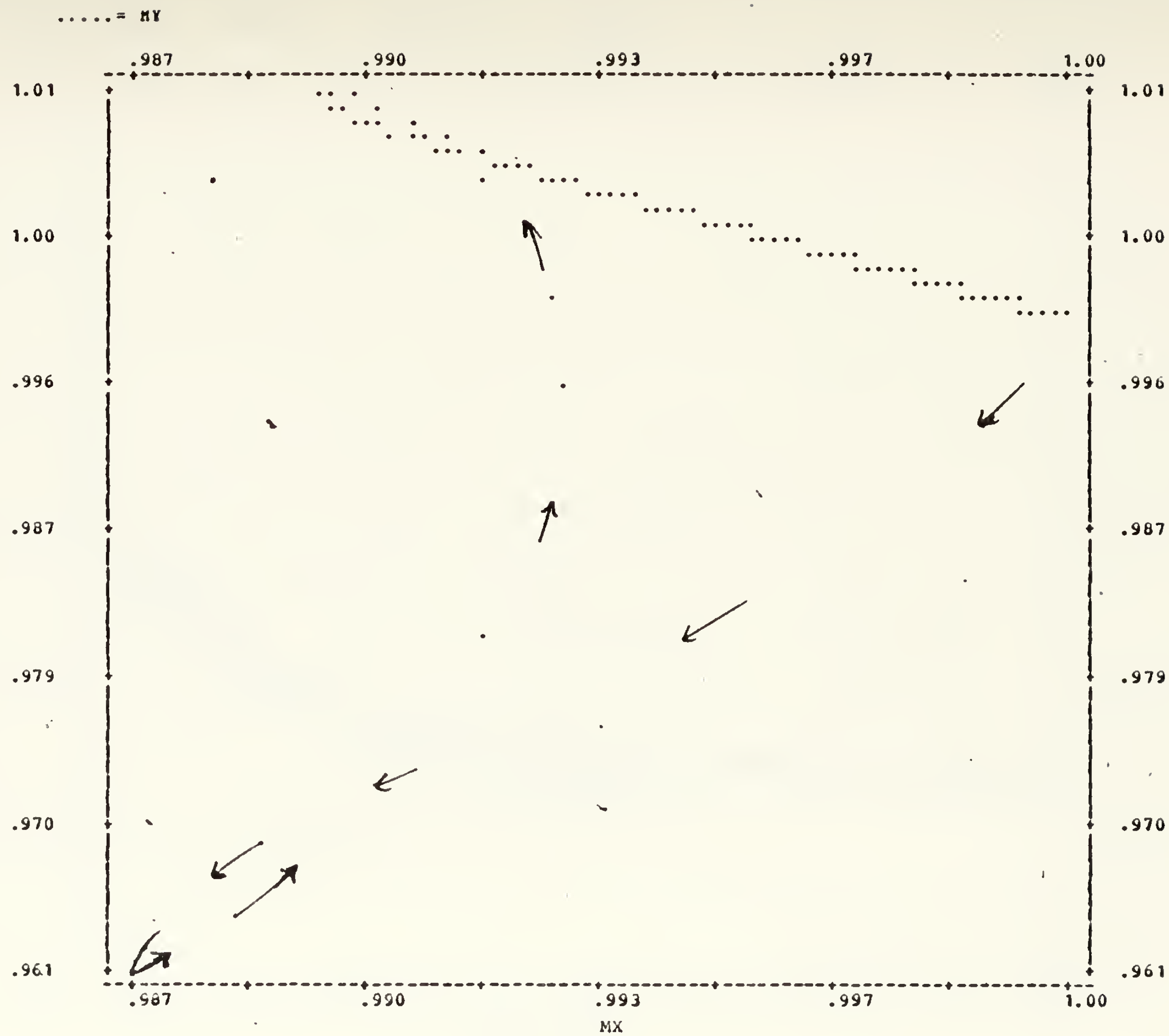


Figure 10. Double Perturbation, stable phase plot ($I_x$ and $I_y$ at 1.25).

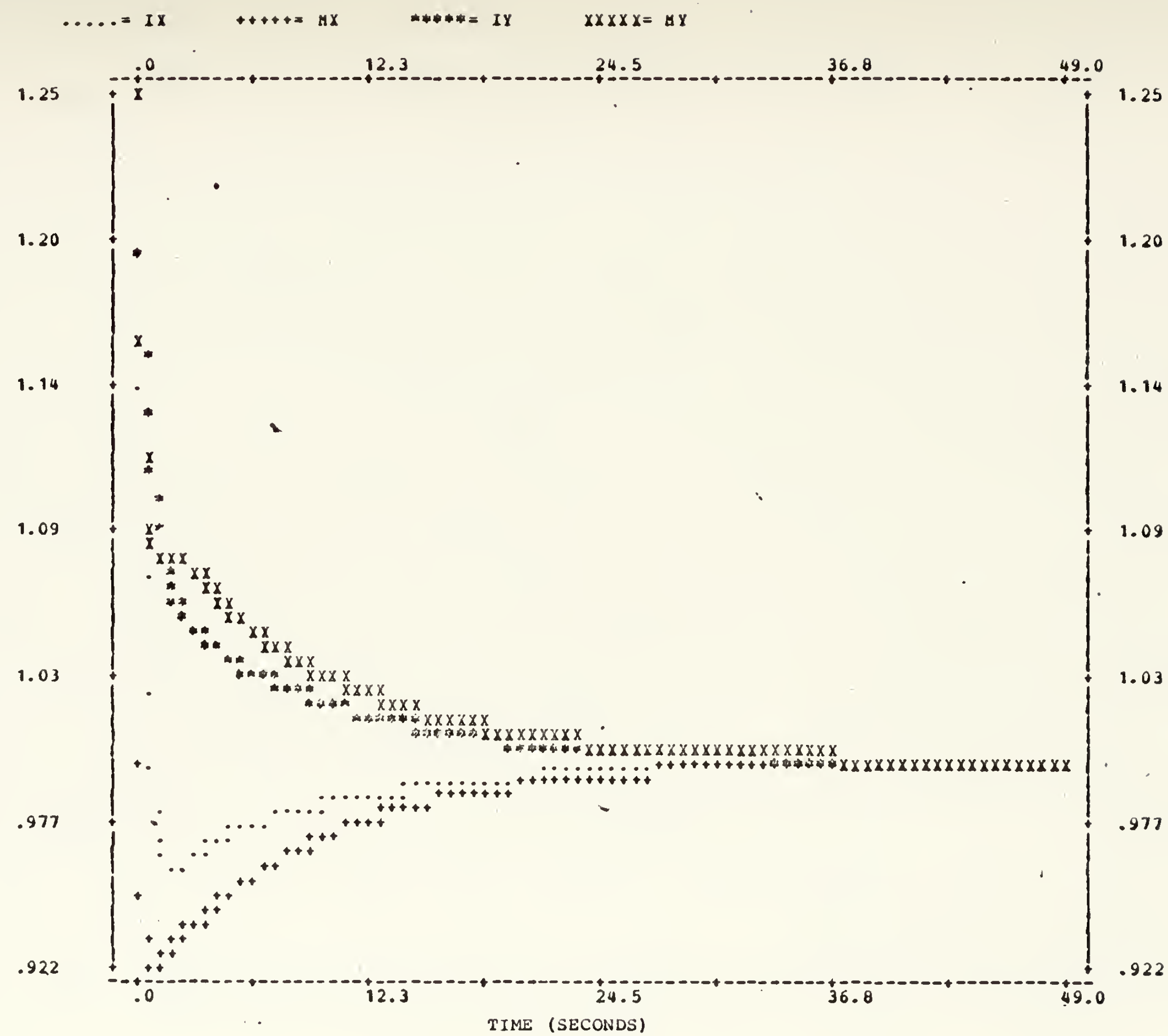


Figure 11. Triple Perturbation, stable ($I_x$, $I_y$ and $M_y$ at 1.25).

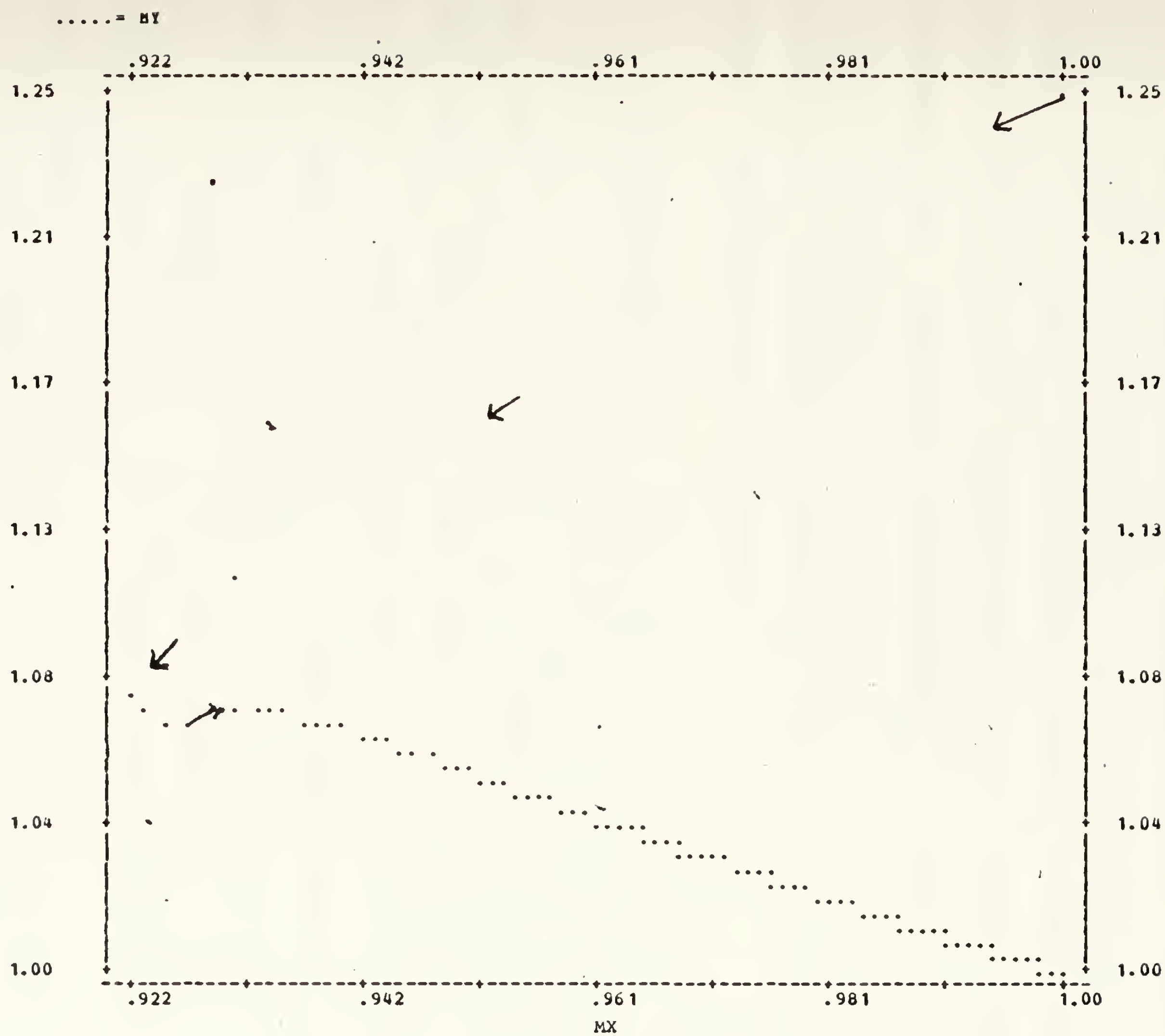


Figure 12. Triple Perturbation, stable phase plot ($I_x$, $I_y$ and $M_y$ at 1.25).

## C. STABILITY WITH $d_{yx}$ AT 0.10

Similar to Table IV, Table V lists the initial parameters for investigation of the model with $d_{yx}$ at 0.10 and $d_{xy}$ at 1.00. Graphically, for a single perturbation, this form of the model produces similar results as before. The exception is given in Figure 13 where only $I_y$ is perturbed (trial 3). $M_x$ and $M_y$ can be seen to return to equilibrium without oscillation as had been indicated in the previous investigation. Again all trials exhibit the dependent variables damping out in the time allowed and since no significantly differently graphical forms were produced they are not included in this section.

## D. INSTABILITY WITH $d_{xy}$ AT 1.40

Table VI lists the initial parameters for the unstable case where $d_{xy} = 1.40$ and $d_{yx}$ is held at 1.00. The additional column on this table represents which force term $M_x$ or $M_y$ is increasing without bound while the other term is decreasing. It should be noted that some variables were given initial values of 1.001 to get comparative plots over time, i.e. using 1.25 as an initial parameter in these cases caused either $M_x$ or $M_y$ to increase or decrease severely immediately.

Graphically three general forms of instability were demonstrated where only one dependent variable was perturbed. These are presented in Figures 14, 15, and 16 (trials 1

## TABLE V

INITIAL CONDITIONS WITH
$d_{xy}$ AT 1.0 AND $d_{yx}$ AT 0.1 (STABLE)

| Trial # | Time | Increment | $I_x$ | $M_x$ | $I_y$ | $M_y$ | Time for 90% Damp Out (Sec) |
|---|---|---|---|---|---|---|---|
| 1 | 0 to 50 | 0.05 | 1.25 | 1.00 | 1.00 | 1.00 | 2.0 |
| 2 | 0 to 50 | 0.05 | 1.00 | 1.25 | 1.00 | 1.00 | 21.0 |
| 3 | 0 to 50 | 0.05 | 1.00 | 1.00 | 1.25 | 1.00 | 21.0 |
| 4 | 0 to 50 | 0.05 | 1.00 | 1.00 | 1.00 | 1.25 | 19.0 |
| 5 | 0 to 50 | 0.05 | 1.25 | 1.25 | 1.00 | 1.00 | 21.0 |
| 6 | 0 to 50 | 0.05 | 1.25 | 1.00 | 1.25 | 1.00 | 20.0 |
| 7 | 0 to 50 | 0.05 | 1.25 | 1.00 | 1.00 | 1.25 | 18.0 |
| 8 | 0 to 50 | 0.05 | 1.00 | 1.25 | 1.00 | 1.25 | 4.0 |
| 9 | 0 to 50 | 0.05 | 1.00 | 1.00 | 1.25 | 1.25 | 34.0 |
| 10 | 0 to 50 | 0.05 | 1.00 | 1.25 | 1.25 | 1.00 | 6.0 |
| 11 | 0 to 50 | 0.05 | 1.25 | 1.25 | 1.25 | 1.00 | 8.0 |
| 12 | 0 to 50 | 0.05 | 1.25 | 1.25 | 1.00 | 1.25 | 6.5 |
| 13 | 0 to 50 | 0.05 | 1.25 | 1.00 | 1.25 | 1.25 | 33.0 |
| 14 | 0 to 50 | 0.05 | 1.00 | 1.25 | 1.25 | 1.25 | 12.5 |

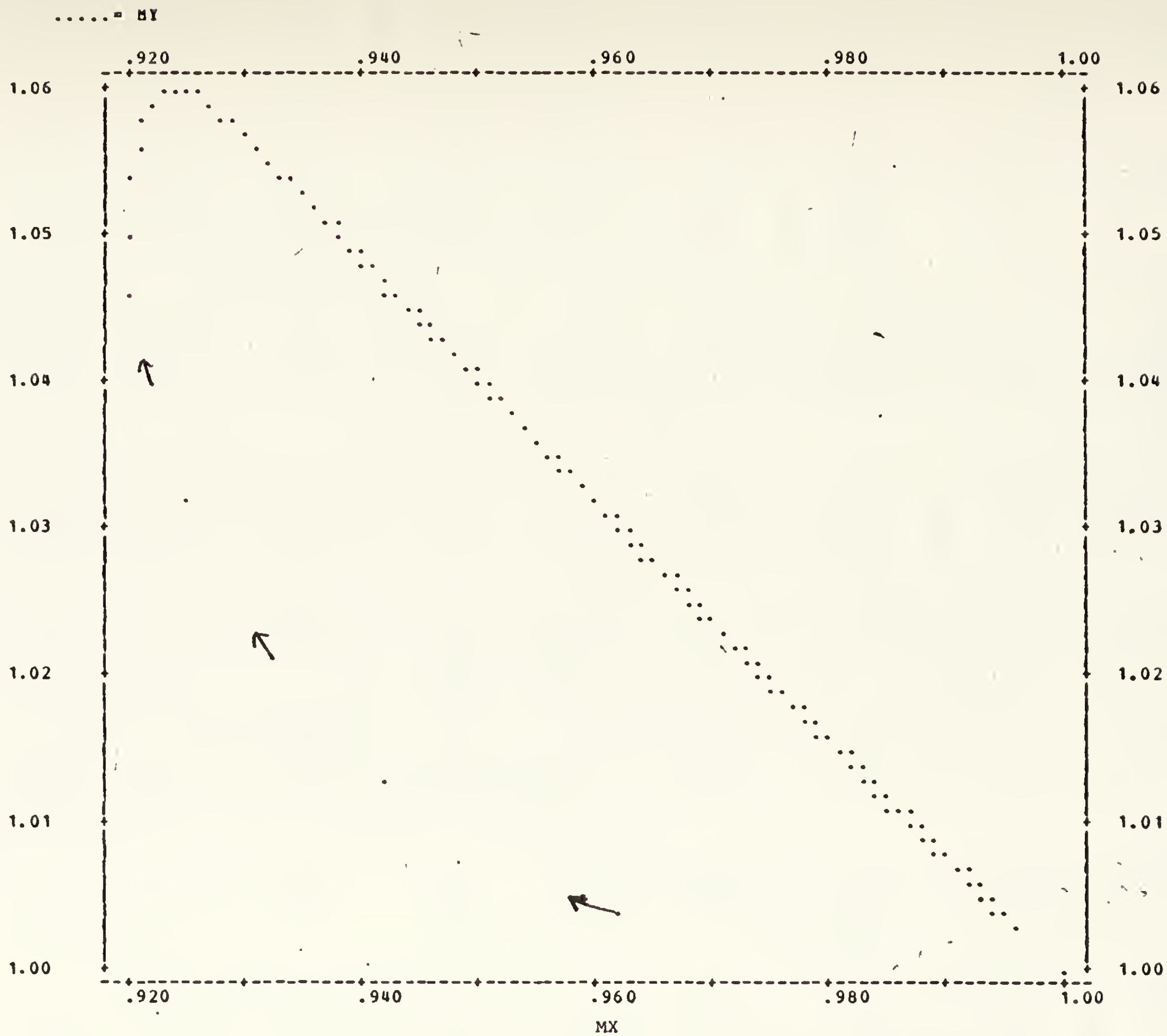


Figure 13. Single Perturbation, stable phase plot ($I_y$ at 1.25).

TABLE VI

INITIAL CONDITIONS WITH
$d_{xy}$ AT 1.4 AND $d_{yx}$ AT 1.0 (UNSTABLE)

| Trial # | Time | Increment | $I_x$ | $M_x$ | $I_y$ | $M_y$ | Increasing Force Term |
|---|---|---|---|---|---|---|---|
| 1 | 0 to 50 | .05 | 1.25 | 1.00 | 1.00 | 1.00 | $M_x$ |
| 2 | 0 to 50 | .05 | 1.00 | 1.25 | 1.00 | 1.00 | $M_x$ |
| 3 | 0 to 50 | .05 | 1.00 | 1.00 | 1.001 | 1.00 | $M_y$ |
| 4 | 0 to 50 | .05 | 1.00 | 1.00 | 1.00 | 1.001 | $M_y$ |
| 5 | 0 to 50 | .05 | 1.25 | 1.25 | 1.00 | 1.00 | $M_x$ |
| 6 | 0 to 50 | .05 | 1.001 | 1.00 | 1.001 | 1.00 | $M_y$ |
| 7 | 0 to 50 | .05 | 1.001 | 1.00 | 1.00 | 1.001 | $M_y$ |
| 8 | 0 to 50 | .05 | 1.00 | 1.00 | 1.001 | 1.001 | $M_y$ |
| 9 | 0 to 50 | .05 | 1.00 | 1.001 | 1.00 | 1.001 | $M_x$ |
| 10 | 0 to 50 | .05 | 1.00 | 1.01 | 1.01 | 1.00 | $M_y$ |
| 11 | 0 to 50 | .05 | 1.25 | 1.25 | 1.25 | 1.00 | $M_x$ |
| 12 | 0 to 50 | .05 | 1.25 | 1.25 | 1.00 | 1.25 | $M_x$ |
| 13 | 0 to 50 | .05 | 1.001 | 1.000 | 1.001 | 1.001 | $M_y$ |
| 14 | 0 to 50 | .05 | 1.00 | 1.001 | 1.001 | 1.001 | $M_y$ |

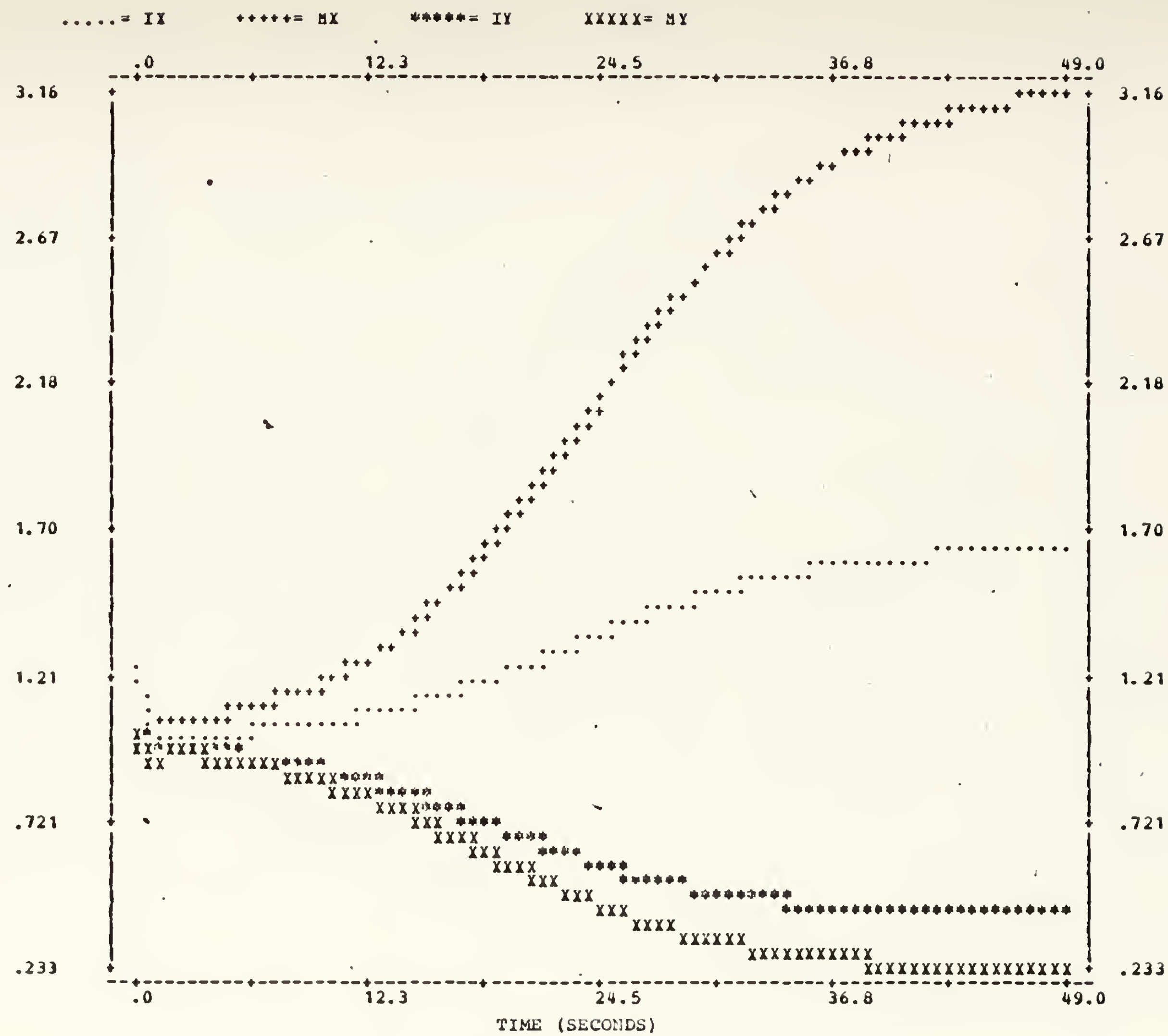


Figure 14. Single Perturbation, unstable ($I_x$ at 1.25).

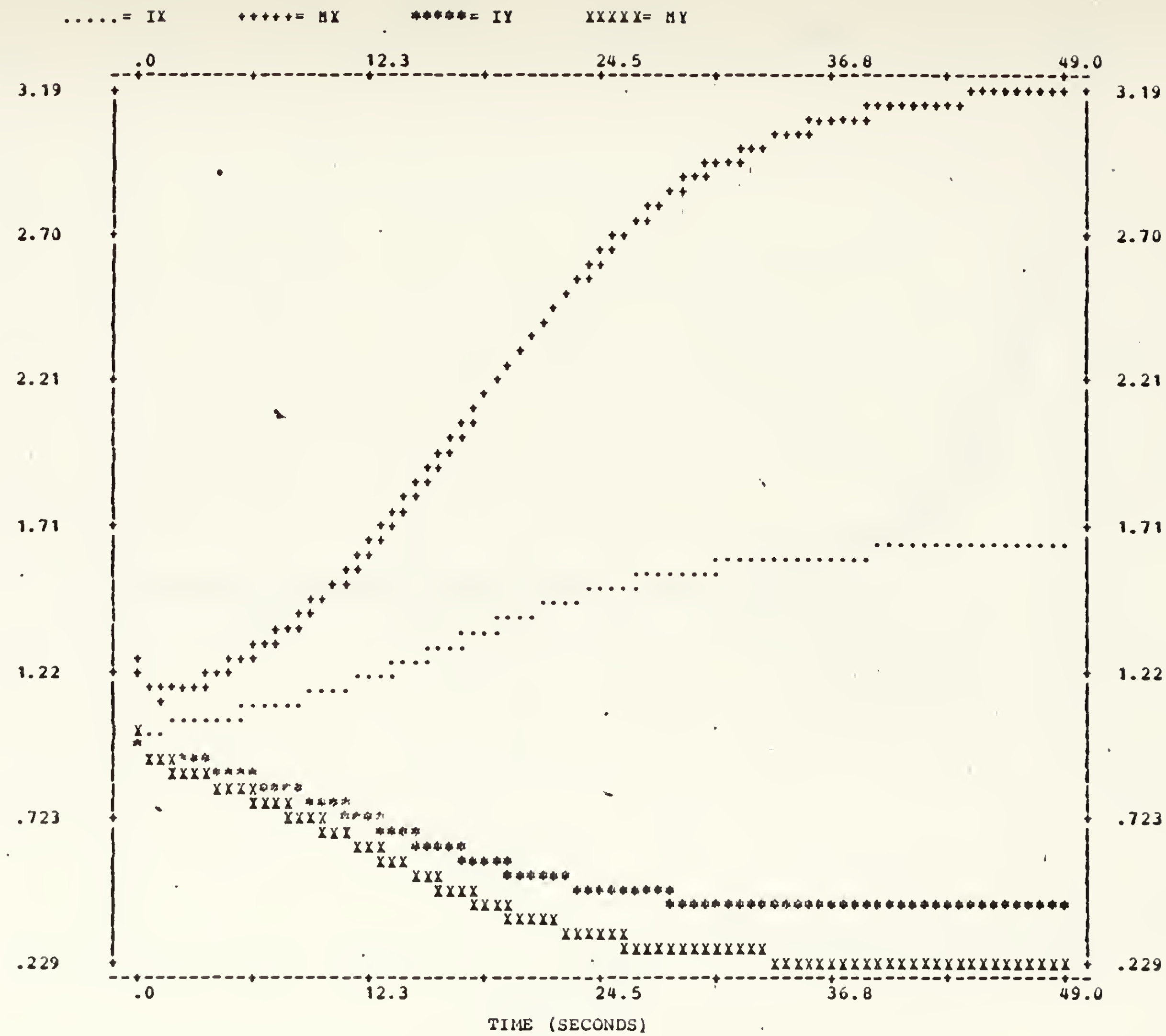


Figure 15. Single Perturbation, unstable ($M_x$ at 1.25).

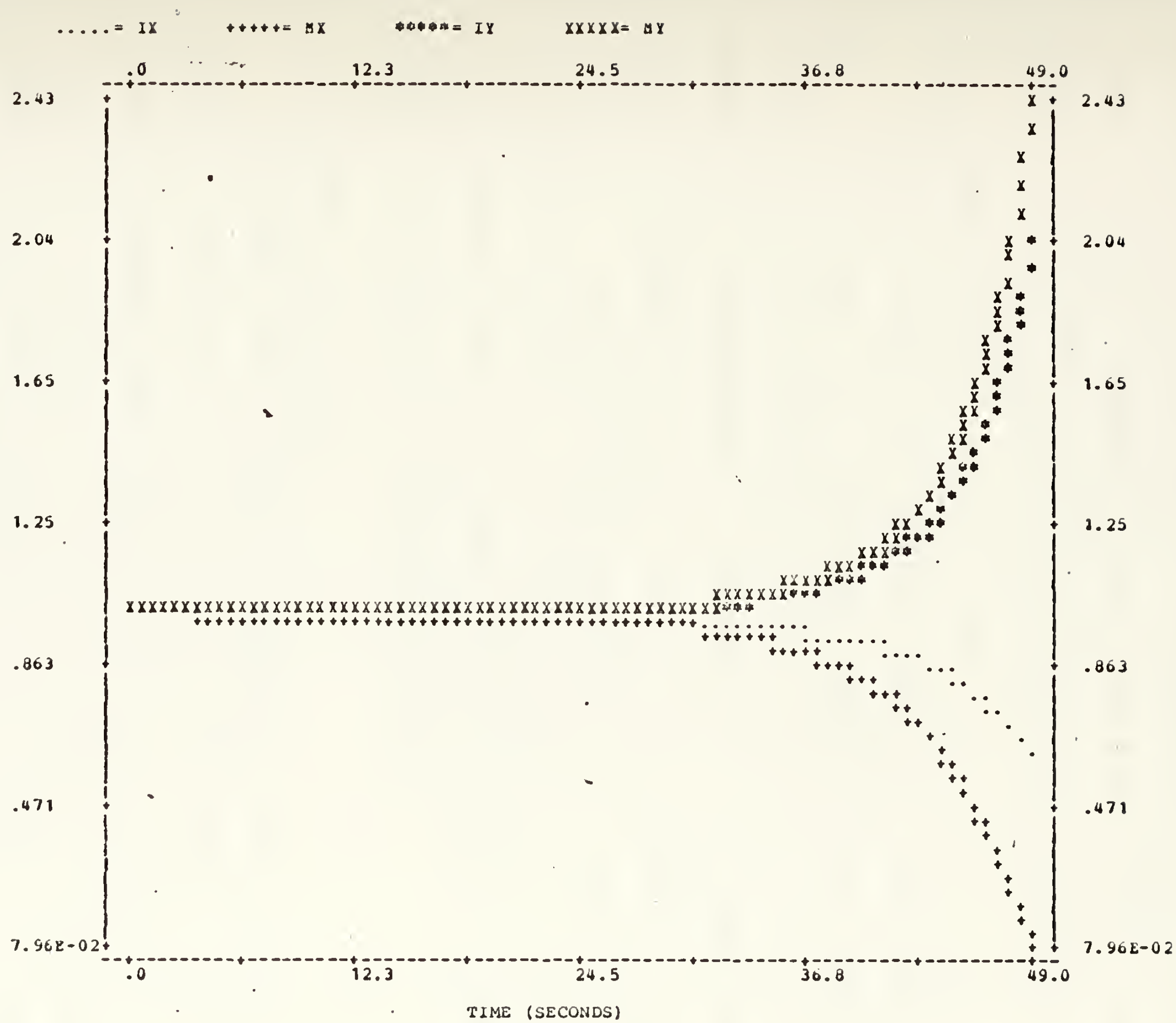


Figure 16. Single Perturbation, unstable ($I_y$ or $M_y$ at 1.001).

through 4). The corresponding phase plots are given in Figures 17, 18, 19, and 20. Specific values for double and triple perturbation did not demonstrate any unusual graphic forms of instability and generally followed those graphic representations displayed for single perturbations.

## E. INSTABILITY WITH $d_{yx}$ AT 1.40

Table VII gives the initial values for trial runs holding $d_{xy}$ at 1.00 and $d_{yx}$ at 1.40. Similar graphic forms were produced in this investigation as those produced in the preceding investigation of instability. Two exceptions are given in Figures 21 and 22 (trial 7) and Figures 23 and 24 (trial 10). It also should be noted that in this case perturbing $M_x$ and $I_y$ together causes $M_x$ to increase. In the previous case perturbing these variables together drove $M_y$ upward, an interesting result.

## F. GENERAL OBSERVATIONS

The cases investigated in sections B and C of this chapter clearly indicate that the Dynamic Model of Modern Military Conflict does exhibit stability as predicted in Chapter 3. Furthermore, it also may behave in an unstable manner as indicated in sections D and E above. A closer look at the unstable cases also reveals bifurcation relative to the values for $d_{xy}$ and $d_{yx}$, initial values for the dependent variables and the resulting increasing term.

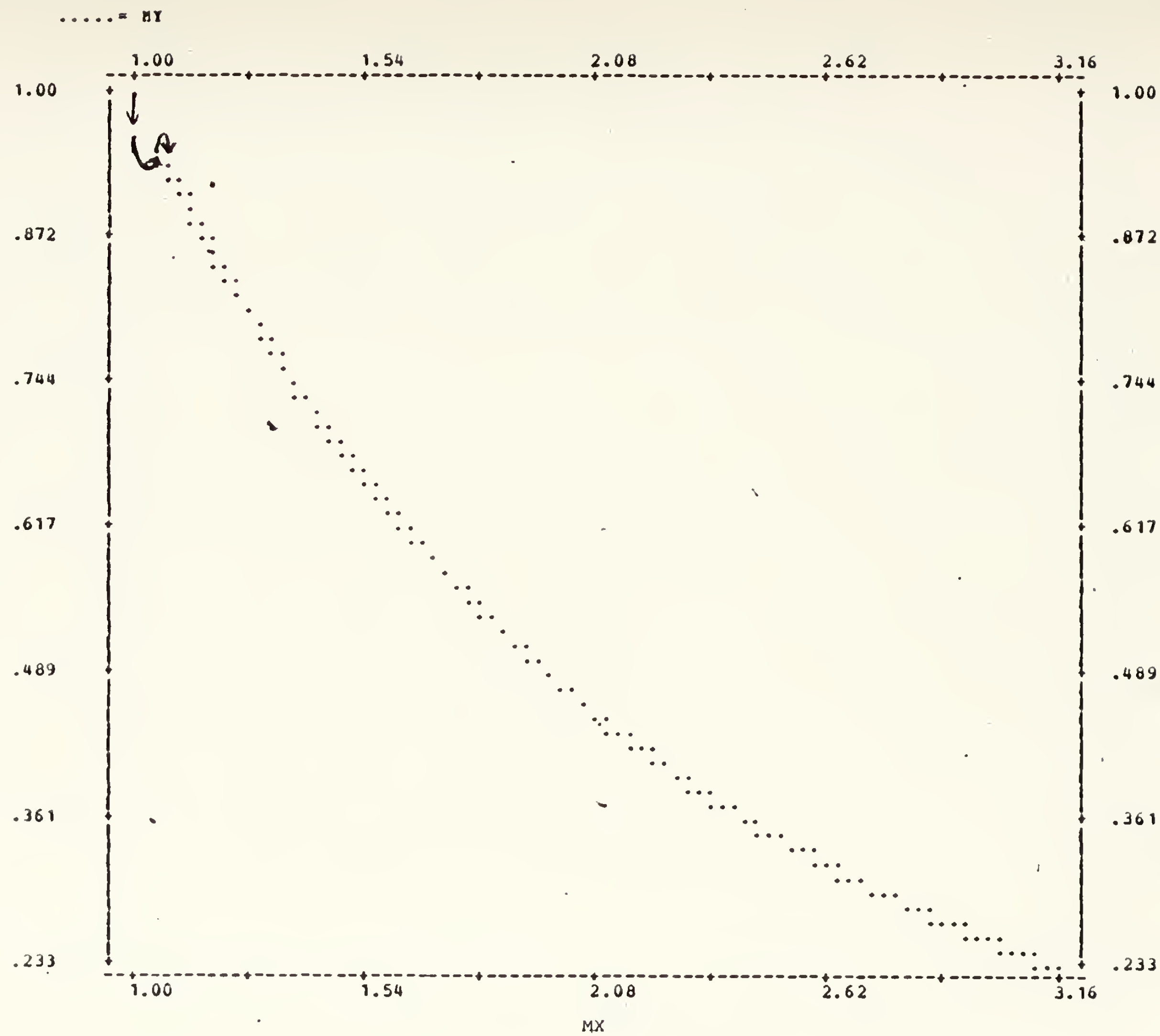


Figure 17. Single Perturbation, unstable phase plot ($I_x$ at 1.25).

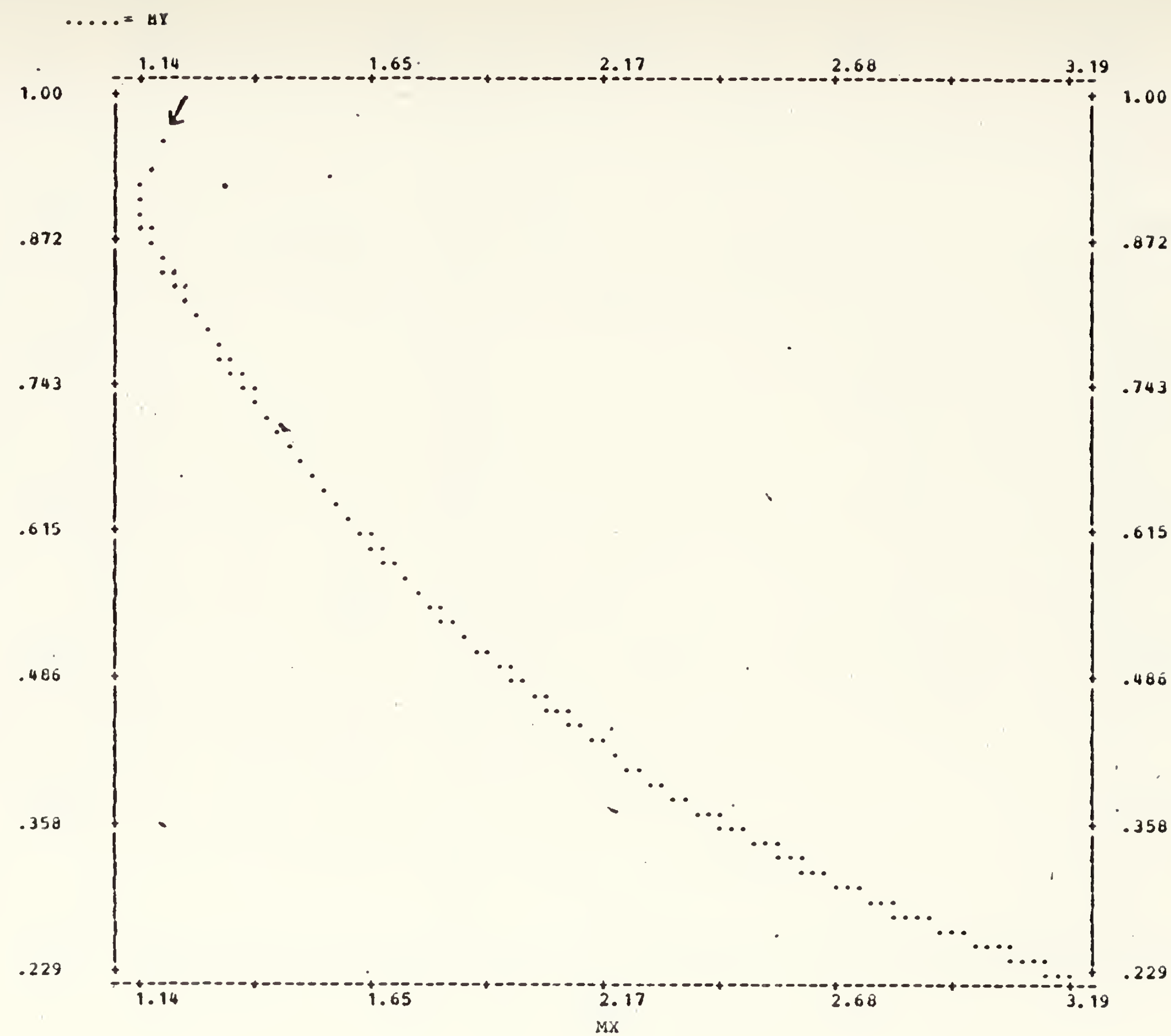


Figure 18. Single Perturbation, unstable phase plot ($M_x$ at 1.25).

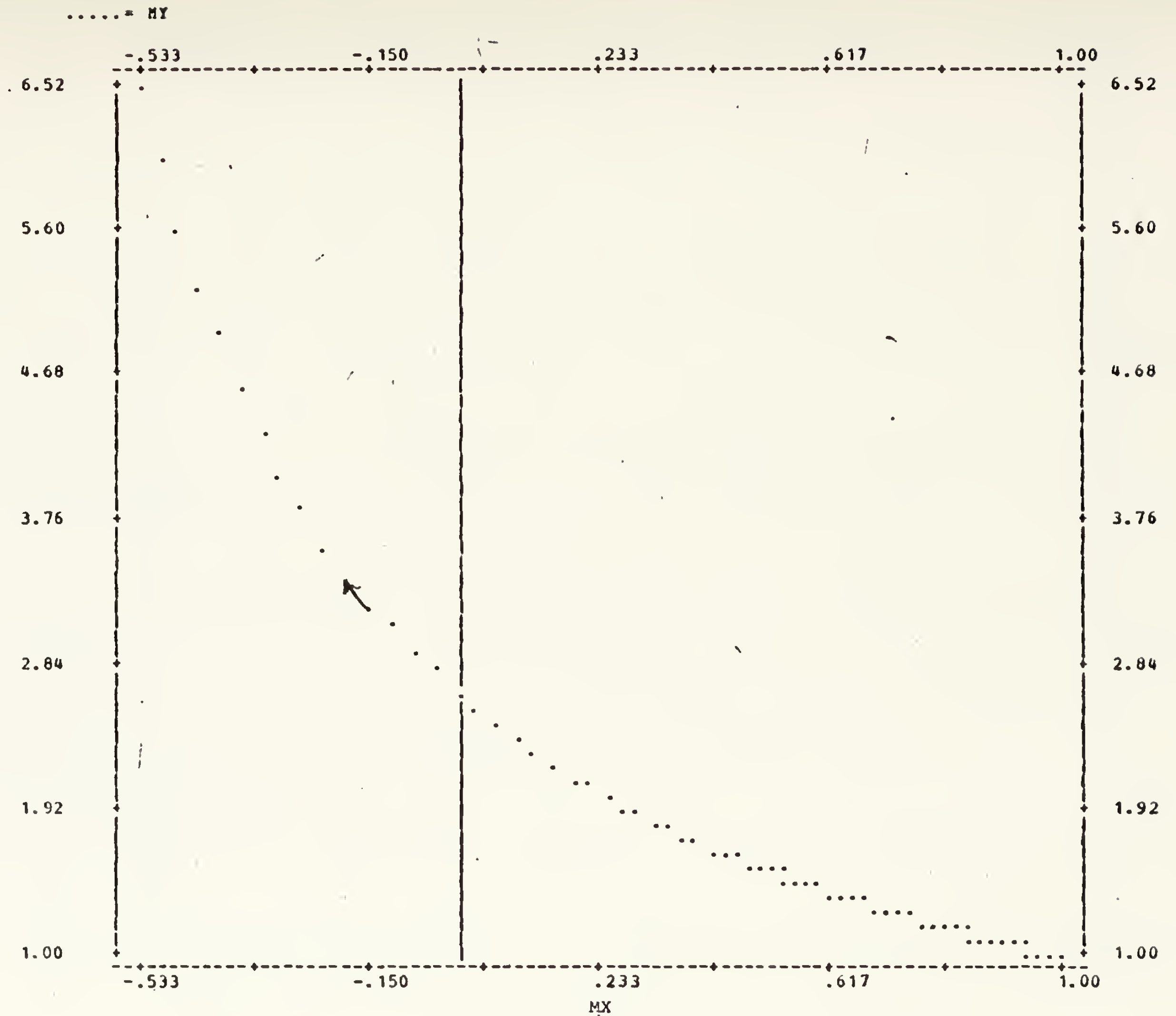


Figure 19. Single Perturbation, unstable phase plot ($I_y$ at 1.001).

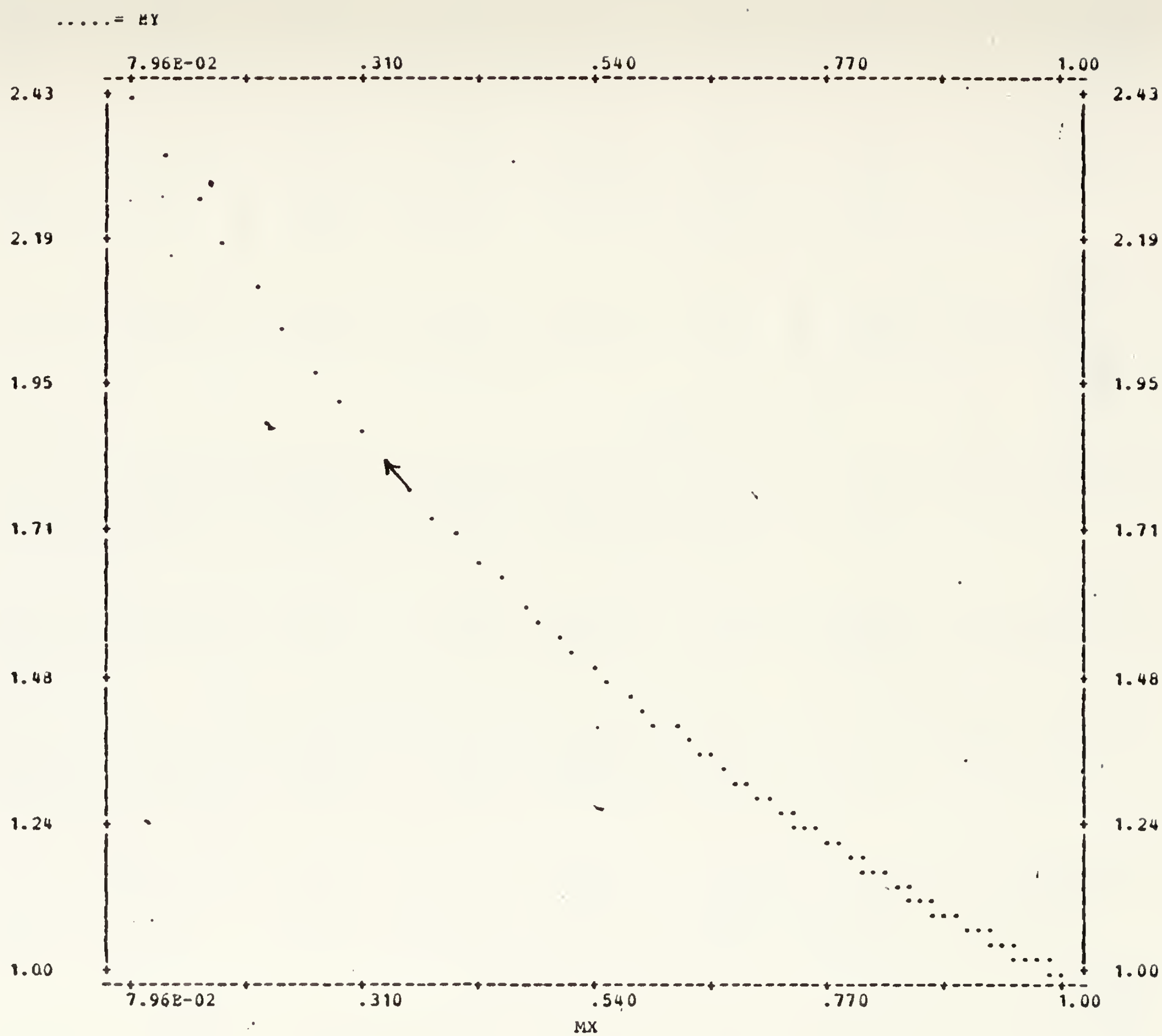


Figure 20. Single Perturbation, unstable phase plot ($M_y$ at 1.001).

TABLE VII

INITIAL CONDITIONS WITH
$d_{xy}$ AT 1.0 AND $d_{yx}$ AT 1.4 (UNSTABLE)

| Trial # | Time | Increment | $I_x$ | $M_x$ | $I_y$ | $M_y$ | Increasing Force Term |
|---|---|---|---|---|---|---|---|
| 1 | 0 to 50 | .05 | 1.25 | 1.00 | 1.00 | 1.00 | $M_x$ |
| 2 | 0 to 50 | .05 | 1.00 | 1.25 | 1.00 | 1.00 | $M_x$ |
| 3 | 0 to 50 | .05 | 1.00 | 1.00 | 1.25 | 1.00 | $M_y$ |
| 4 | 0 to 50 | .05 | 1.00 | 1.00 | 1.00 | 1.25 | $M_y$ |
| 5 | 0 to 50 | .05 | 1.25 | 1.25 | 1.00 | 1.00 | $M_x$ |
| 6 | 0 to 50 | .05 | 1.01 | 1.00 | 1.01 | 1.00 | $M_y$ |
| 7 | 0 to 50 | .05 | 1.01 | 1.00 | 1.00 | 1.01 | $M_y$ |
| 8 | 0 to 50 | .05 | 1.00 | 1.00 | 1.01 | 1.01 | $M_y$ |
| 9 | 0 to 50 | .05 | 1.00 | 1.01 | 1.00 | 1.01 | $M_x$ |
| 10 | 0 to 50 | .05 | 1.00 | 1.01 | 1.01 | 1.00 | $M_x$ |
| 11 | 0 to 50 | .05 | 1.25 | 1.25 | 1.25 | 1.00 | $M_x$ |
| 12 | 0 to 50 | .05 | 1.25 | 1.25 | 1.00 | 1.25 | $M_x$ |
| 13 | 0 to 50 | .05 | 1.001 | 1.00 | 1.001 | 1.001 | $M_y$ |
| 14 | 0 to 50 | .05 | 1.00 | 1.001 | 1.001 | 1.001 | $M_y$ |

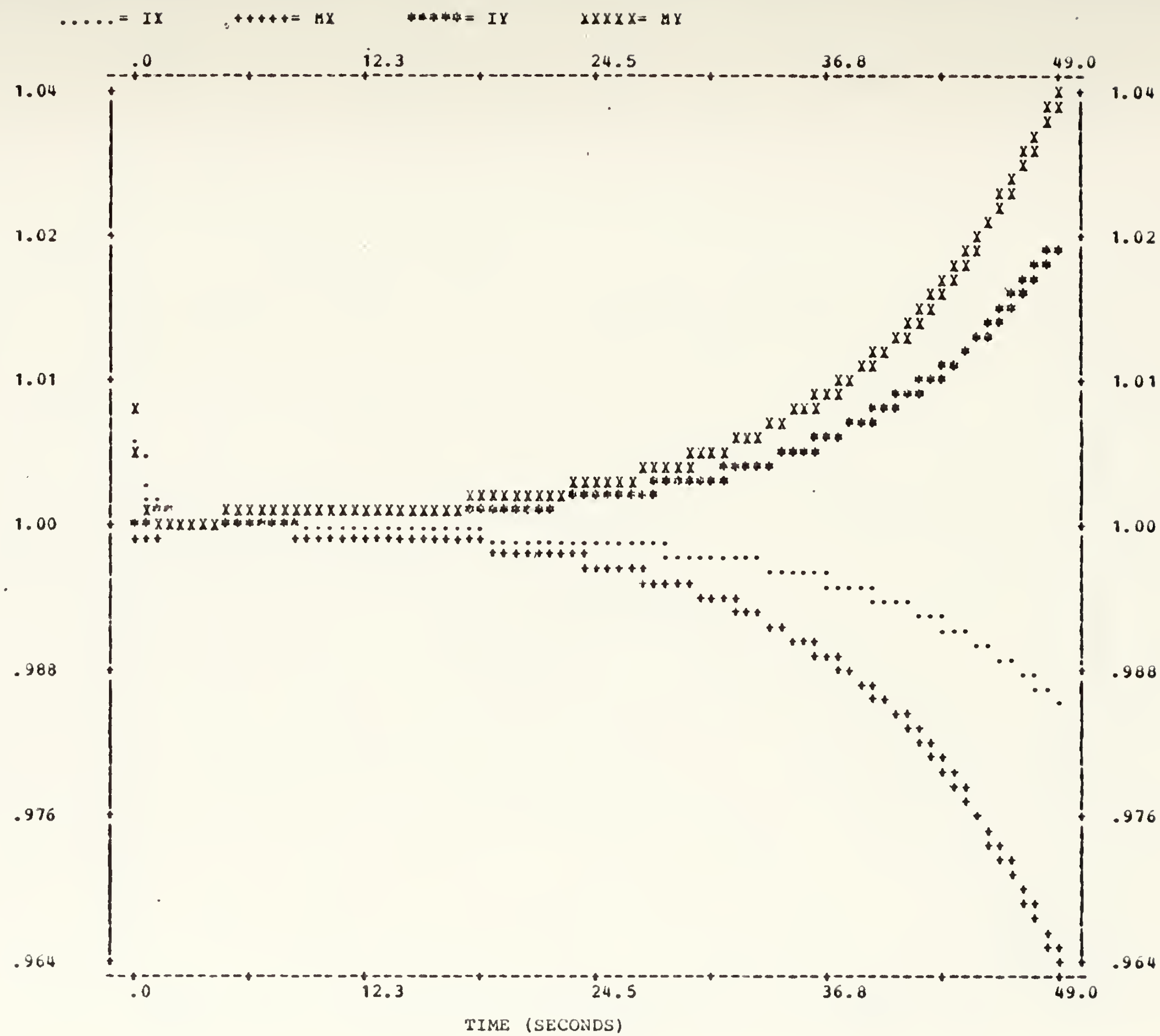


Figure 21. Double Perturbation, unstable ($I_x$ and $M_y$ at 1.01).

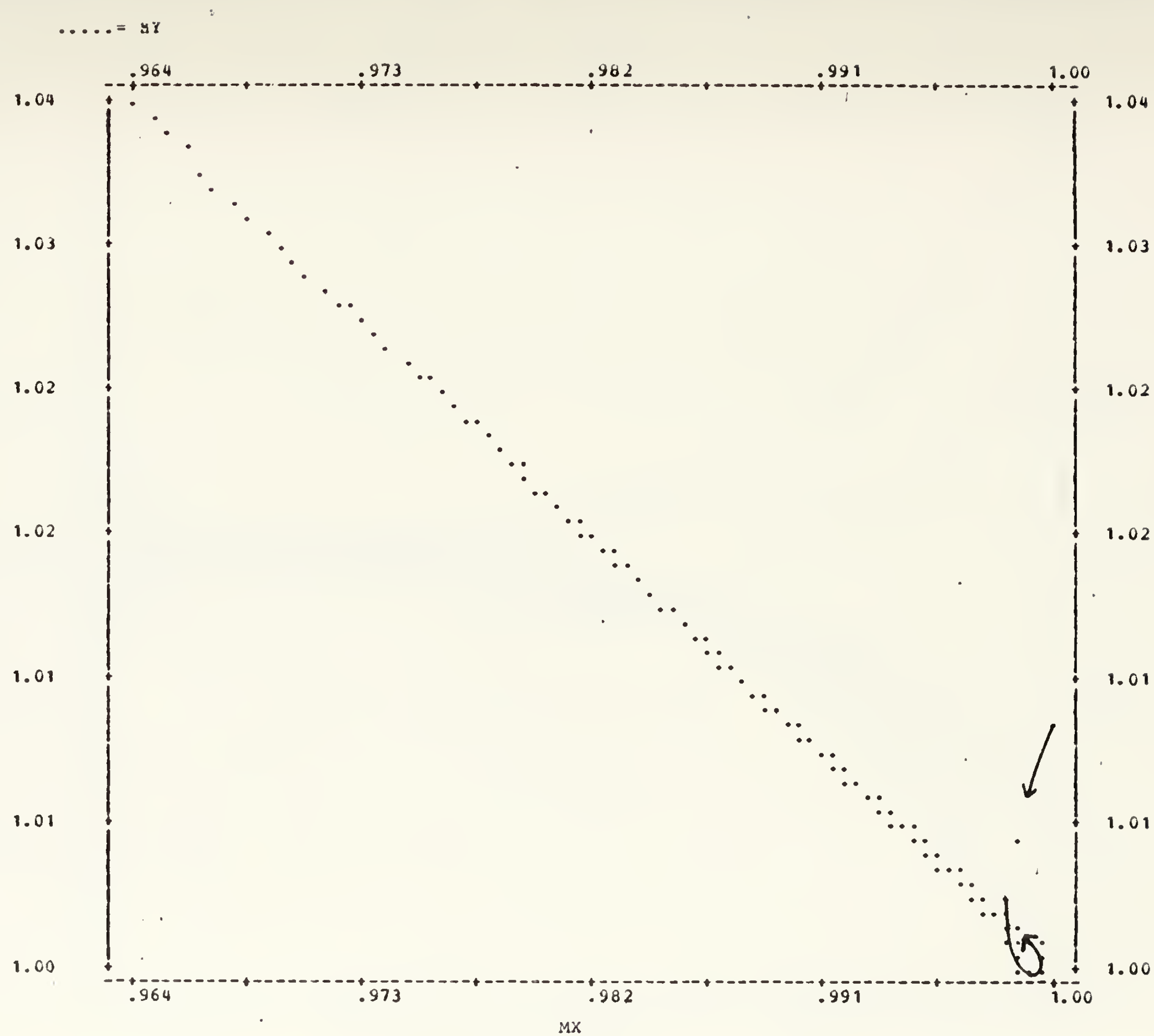


Figure 22. Double Perturbation, unstable phase plot ($I_x$ and $M_y$ at 1.01).

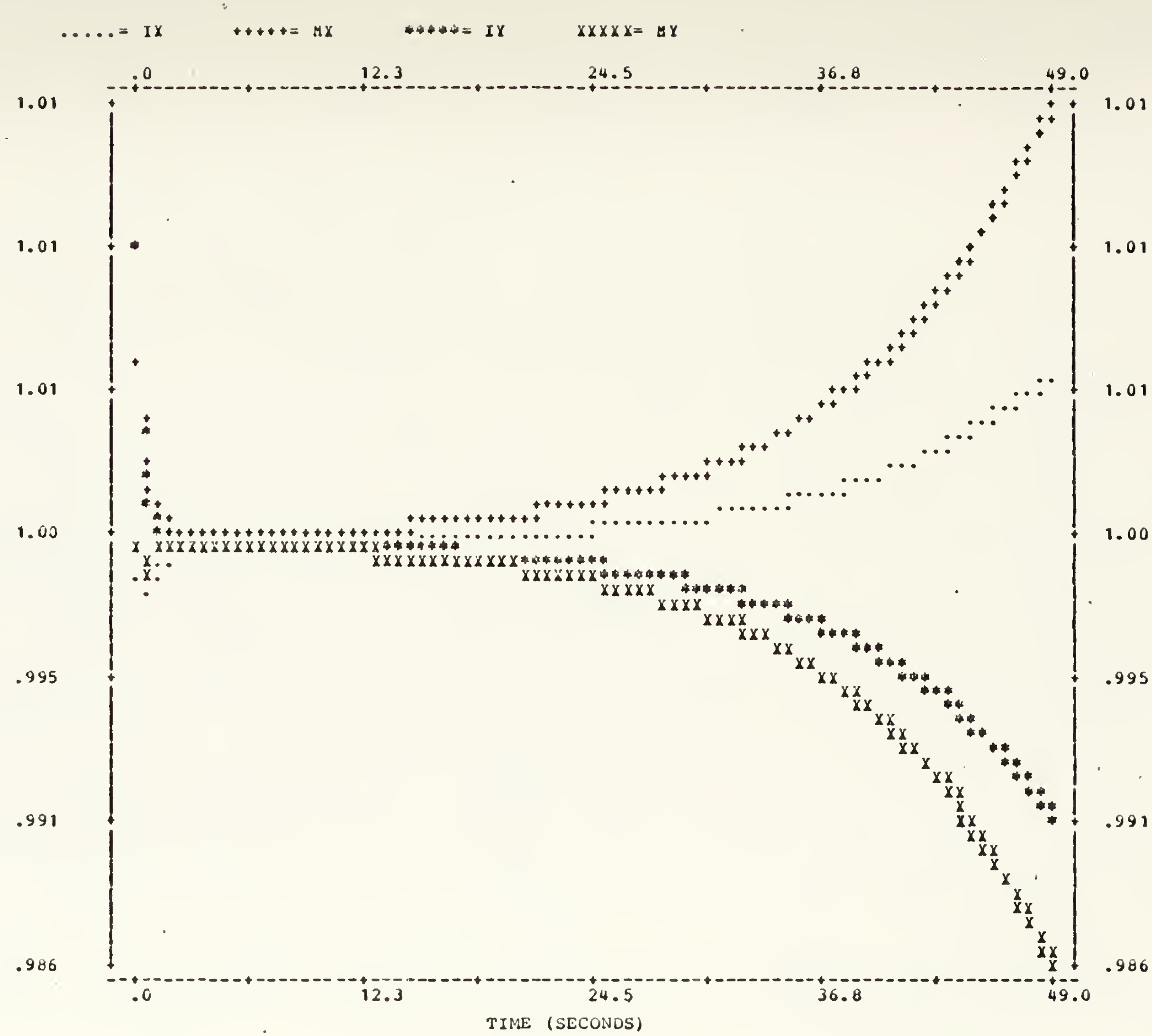


Figure 23. Double Perturbation, unstable ($M_x$ and $I_y$ at 1.01).

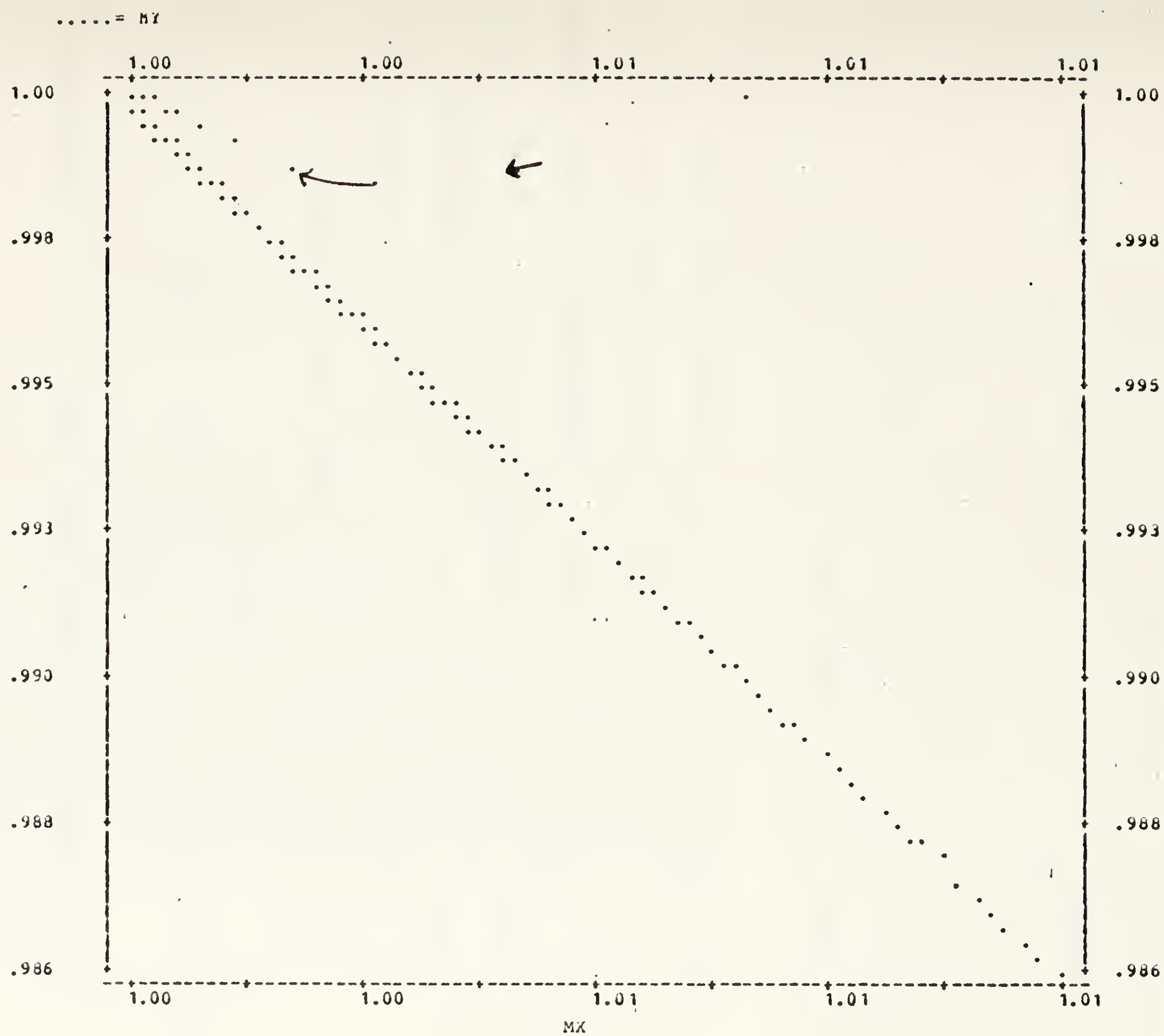


Figure 24. Double Perturbation, unstable phase plot ($M_x$ and $I_y$ at 1.01).

A simple display of this bifurcation is seen when only one element (dependent variable) is perturbed as in trials 1 through 4 in Table VI and VII. Relative to Table VI, $d_{xy}$ is set at 1.40 and $d_{yx}$ at 1.00. This gives Y an advantage over X in $C^3$ effectiveness, however, it can be seen that in trials 1 and 2 of this table, X can overwhelm Y ($M_x$ increasing, $M_y$ decreasing) if given an advantage in either information ($I_x$) or forces ($M_x$). Similarly, relative to Table VII with $d_{xy}$ set at 1.00 and $d_{yx}$ at 1.40, Y can be seen to overwhelm X in trials 3 and 4. Evidences of bifurcation are a bit more complicated in the remaining trials of these tables.

In Table VI, trials 5, 9, 11, and 12 exhibit this bifurcation where y has an advantage in $C^3$ but is overwhelmed by X. Similarly in Table VII, trials 6, 7, 8, 13 and 14 show X being overwhelmed by Y, even though it has a $C^3$ advantage. Trail 10 is not a bifurcation example, but as mentioned in section E, does produce an interesting result.

Specifically, from Table VI, a slight advantage in $M_x$ and $I_y$ results in Y overwhelming X when Y has a $C^3$ advantage. Whereas from Table VII, the slight advantage in $M_x$ and $I_y$ results in X overwhelming Y when X has the $C^3$ advantage. This seems consistent with common sense.

# V. EQUILIBRIUM ANALYSIS

All the preceding analysis of stability was predicated upon a constant equilibrium vector of $\underset{\sim}{1}$ for ease of calculation. Another question that should be answered at this point is "what happens to equilibrium values if, for instance, when replenish rates $(Q_x, R_x, Q_y, R_y)$ are kept constant and $d_{xy}$ and/or $d_{yx}$ is allowed to vary?" To answer this question the equilibrium equations and initial analysis conditions are brought back into the analysis.

## A. METHOD OF ANALYSIS

First, the original equilibrium equations are set to 0 and solved for the replenishment rates. This result is then used to solve the same set of equations for equilibrium as a function of $d_{xy}$ and $d_{yx}$.

## B. EQUILIBRIUM RELATION TO $d_{xy}$ AND $d_{yx}$

Each equilibrium equation is set to 0 and solved for replenishment rates.

$$C_{xx}M_{xe} - \alpha_{xy}I_{ye} - \gamma_{xy}M_{ye} - A_x + Q_x = 0$$

$$0(1) - (1)(1) - 0(1) - 0.5 + Q_x = 0$$

$$Q_x = 1.5 \qquad \text{(Eqn 5.1)}$$

$$-d_{xx}I_{xe} - (\delta_{xy}+d_{xy})I_{ye} - (\beta_{xy} + b_{xy})M_{ye} - B_x + R_x = 0$$

$$0(1) - (0 + d_{xy})\,1 - (1 + 0)\,1 - 0.5 + R_x = 0$$

$$R_x = d_{xy} + 1.5 \qquad \text{(Eqn 5.2)}$$

$$-\alpha_{yx}I_{xe} - \gamma_{yx}M_{xe} + C_{yy}\ M_{ye} - A_y + Q_y = 0$$

$$0\ (1) - 1\ (1) + 0\ \ (1) - 0.5 + Q_y = 0$$

$$Q_y = 1.5 \qquad \text{(Eqn 5.3)}$$

$$-(\delta_{yx} + d_{yx})I_{xe} - (\beta_{yx} + b_{yx})M_{xe} - d_{yy}I_{ye} - B_y + R_y = 0$$

$$-\ (0 + d_{yx})1 - (1 + 0)1 - 0\ (1) - 0.5 + R_y = 0$$

$$R_y = d_{yx} + 1.5 \qquad \text{(Eqn 5.4)}$$

Utilizing these results the same equations are solved again for the equilibrium points starting with some arbitrary initial values for $d_{xy}$ and $d_{yx}$, $d'_{xy}$ and $d'_{yx}$ respectively to fix the replenishment rates.

$$C_{xx}M_{xe} - \alpha_{xy}I_{ye} - \gamma_{xy}M_{ye} - A_x + Q_x = 0$$

$$0\ (M_{xe}) - (1)(I_{ye}) - 0(M_{ye}) - 0.5 + Q_x = 0$$

$$I_{ye} = Q_x - 0.5$$

Using the results of equation 5.1

$$I_{ye} = 1 \qquad \text{(Eqn 5.5)}$$

$$-d_{xx}I_{xe} - (\delta_{xy} + d_{xy})I_{ye} - (\beta_{xy} + b_{xy})M_{ye} - \beta_x + R_x = 0$$

$$-(0)I_{xe} - (0 + d_{xy})I_{ye} - (\beta_{xy} + 0)\ M_{ye} - 0.5 + R_x = 0$$

$$M_{ye} = -d_{xy}I_{ye} - 0.5 + R_x$$

Using the result of equations 5.2 and 5.5 and substituting $d'_{xy} + 1.5$ for $R_x$

$$M_{ye} = -d_{xy} - 0.5 + (d_{xy}' + 1.5)$$

$$M_{ye} = 1.0 - (d_{xy} - d_{xy}') \qquad \text{(Eqn 5.6)}$$

$$-\alpha_{yx}I_{xe} - \gamma_{yx} M_{xe} + C_{yy}M_{ye} - A_y + Q_y = 0$$

$$-(0)I_{xe} - (1)M_{xe} + (0)M_{ye} - 0.5 + Q_y = 0$$

$$M_{xe} = Q_y - 0.5$$

Using the results of equation 5.3

$$M_{xe} = 1 \qquad \text{(Eqn 5.7)}$$

$$-(\delta_{yx} + d_{yx})I_{xe} - (\beta_{yx} + b_{yx})M_{xe} - d_{yy}I_{ye} - B_y + R_y = 0$$

$$-(0 + d_{yx})I_{xe} - (1 + 0)M_{xe} - (0)I_{ye} - 0.5 + R_y = 0$$

Using the results of equation 5.7

$$I_{xe} = \frac{1.5 - R_y}{-d_{yx}}$$

Using the results of equation 5.4 and substituting

$d_{yx}' + 1.5$ for $R_y$

$$I_{xe} = \frac{d'_{yx} + 1.5 - 1.5}{d_{yx}}$$

Adding $d_{yx} - d_{yx}$ to the numerator

$$I_{xe} = 1 - \frac{(d_{yx} - d'_{yx})}{d_{yx}}$$

C. ASSIMILATION

It suffices to say then that $I_{ye}$ and $M_{xe}$ are not effected by changes in $d_{xy}$ or $d_{yx}$. On the other hand $M_{ye}$ varies in the opposite direction to deviations from its nominal value. $I_{xe}$ varies in the opposite direction to deviations from its nominal value normalized by $d_{xy}$.

More specifically, equation 5.6 indicates that force Y would require fewer general purpose resources to hold force X to 1 provided, force Y increased its $C^3$ enhancement beyond its initial value. In a similar fashion equation 5.8 indicates force X would not require as much information to hold force Y to 1 provided force X increased its $C^3$ enhancement beyond its intial value. It should be noted that these equations may not be true in general, but rather may only be true for the specific initial values used in this analysis.

# VI. SUMMARY AND CONCLUSIONS

This thesis has described the initial investigation of the Dynamic Model of Modern Military Conflict. Following a brief description of the model, investigation of stability was undertaken using a method of analysis previously prescribed for the study of deterministic systems such as ecosystems. Through this investigation it has been demonstrated that the model does in fact behave as one might expect, given a set of somewhat reduced initial parameters. In this respect, this research endeavor did not attempt to fully prove or disprove the model but rather its purpose was to permit observation of the model behavior as an initial step toward further study.

## A. GENERAL OBSERVATIONS

It can be seen that the Dynamic Model of Modern Military Conflict, given a set of asymmetrical initial parameters exhibits behavior that corresponds to a status quo (stability) or a dominance of one force, combat force if you will, over another (instability). The significance of this observation lies in the plausible use of this model in predicting outcomes of military conflict. Particularly, the incorporation of terms relating to counter $C^3$ and $C^3$ enhancements along with more traditional terms such as loss rates, replenishment rates and Lanchester coefficients, adds a contemporary flavor

to this model. The actual employment of this model in playing "what if?" type wargames however cannot be undertaken as yet without additional research including the remaining terms of this model.

## B. IMPLICATIONS FOR FUTURE RESEARCH

Specifically, the following areas for additional research are suggested based on the initial investigation described in this thesis:

1. Location of other equilibrium points besides the vector 1 utilized herein.

2. Determination of an appropriate Lyapunov function suitable for predicting global stability.

3. Prediction of bifurcation through an analytical approach.

4. Further investigation of model behavior relative to variations of other initial parameters (i.e. counter $C^3$ coefficients).

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